Exercise 7.0

THE CHANGING DIURNAL CIRCLES OF THE SUN

I. The Apparent Annual Motion of the Sun

A star always rises and sets at the same place on the horizon and, hence, it is above the horizon for the same amount of time every day of the year. That is, a star traces out the same diurnal circle every day because its declination does not change. This is not true for the Sun, Moon, and planets. Near mid-latitude in the summer, the Sun rises in the northeast and sets in the northwest, with the daylight portion of the day longer than the nighttime portion. During the winter, the Sun rises in the southeast and set in the southwest, with the amount of daylight hours less than the amount of nighttime hours. Twice a year the Sun rises exactly at the East Point of the horizon and sets exactly at the West Point. On these two days, there are twelve hours of sunlight and twelve hours of darkness. These two events occur about March 21 and September 22 and the Sun is then located at the **Vernal** or **Autumnal Equinox** points in the sky. The day of the year with the longest daylight period occurs about June 21, when the Sun is at the **Summer Solstice**. The day of the year with the shortest amount of daylight occurs about December 22, when the Sun is at the **Winter Solstice**.   These things are related to the fact that the diurnal circle of the Sun slowly changes from one day to the next, which means the declination of the Sun changes slowly throughout the year
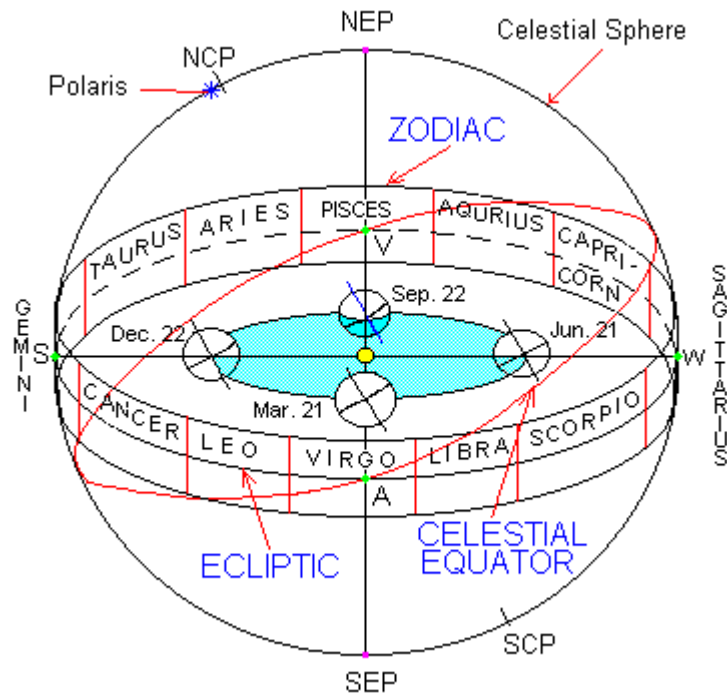

Figure 1. The relationship between the plane of the Earth's orbit, the ecliptic, the Zodiac, and the celestial equator.   The position of the Earth in its orbit around the Sun is shown for four important seasonal dates of the year.

The changing declination and right ascension of the Sun is caused by the fact that Earth revolves in orbit around the Sun with its axis of rotation tilted with respect to the plane of its orbit. The amount of this tilt is 23.5^o as measured from a line perpendicular to the Earth's orbital plane. This line pierces the celestial sphere at two points called the **north ecliptic pole and south ecliptic pole.** In Figure 1, these two points are designated as **NEP** and **SEP.**

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As the Earth revolves in orbit, its axis of rotation remains parallel to itself. That is, the axis always points towards the north celestial pole (**NCP**). For an observer on the Earth, the Sun appears to move in a circle around the celestial sphere eastward with respect to the **fixed stars.** This circle is called the **ecliptic** and is labeled VSAW in Fig. 1. Note that the ecliptic lies in the plane of the Earth's orbit. Thus the ecliptic is inclined to the celestial equator by 23.5°. This angle is referred to as the **obliquity of the ecliptic**. Therefore, as the Sun appears to move around the ecliptic over the course of a year, its angular distance from the celestial equator (declination) changes. The 2 points of intersection of the ecliptic and the celestial equator are called the **equinoxes**.

Now, the Earth takes 365.2422 days to complete a 360° orbit around the Sun. Therefore:

**In the sky, the Sun appears to move eastward along the ecliptic
about one degree per day, relative to the "fixed stars."**

This apparent motion of the Sun is merely a reflection of the Earth's revolution. On or about March 21, the Sun crosses the celestial equator moving from south to north. The Sun is then at the point labeled V in Figure 1, which is called the vernal equinox. The Sun reaches its maximum northerly declination (+23.5°) on or about June 21. It is then at the point labeled S, which is called the **summer solstice**. The point where the Sun reaches its maximum southerly declination (-23.5°) is the **winter solstice,** W**.**

The equinoxes and solstices are associated with the above dates only in the sense that the Sun is located at these points on or about those dates. Actually these dates may change by a few days from on year to another. The equinoxes and solstices should be thought of as positions on the celestial sphere rather than as specific dates. For example, the vernal equinox is a point on the celestial sphere located in the constellation of stars called Pisces. Like other points on the celestial sphere, an observer on the Earth sees the solstices and equinoxes to execute diurnal circles that depend on the declination of these points. The vernal equinox rises at the east point of the horizon and sets at the west point of the horizon every day throughout the year. Hence, when the Sun is at the vernal equinox, it also rises at the east point and sets at the west point. The daily motion of the Sun is discussed in more detail in the next section.

The following table lists the declinations and right ascensions for some important points in the sky. A "**U**" means undefined. Note that objects on the celestial equator have declinations of zero degrees and that objects south of the celestial equator have negative declinations.

A TABLE OF SOME DECLINATIONS AND RIGHT ASCENSIONS

POINT	DECLINATION (Degrees)	RIGHT ASCENSION (Hours)
VE	0.00	0.00
SS	+23.5	6.00
AE	0.00	12.00
WS	- 23.5	18.00
NCP	+90.00	U
SCP	- 90.00	U

The right ascension and declination of the Sun's position vary as it moves along the ecliptic from one point to another as shown in the above table. The time required for the Sun to cycle through its coordinates, that is, the time it takes the Sun to go once around the ecliptic from the vernal equinox back to the vernal equinox is called the **year of the seasons or tropical year.** The Moon cycles through its coordinates in about one month. The time required for a planet to do this depends on its period of revolution.

Because the orbits of the planets of our Solar System lie nearly in the same plane as the Earth's orbit, the planetary orbits are circles in the sky close to the ecliptic. The belt of the sky,

centered on the ecliptic and where most of the planets are observed, is called the **zodiac.** The ancients organized the stars that are located in this belt into twelve constellations known as the **zodiacal constellations**. The Sun moves from one zodiacal constellation into the next one in about one month. Visualize this in Figure 1.

II. The Daily Motion of the Sun

As was demonstrated in Exercises 3.0:

Diurnal motion takes place along diurnal circles that are centered on the axis of rotation and are parallel to the celestial equator.

On the dates of the equinoxes, when the Sun's declination is 0°, its diurnal circle is coincident with the celestial equator. As seen from the north geographic pole, the Sun would be seen to circle the horizon in the course of the day and not rise or set. At all other latitudes the Sun rises at the east point and sets at the west point of the observer's horizon and there are equal lengths of daylight and darkness since the horizon cuts the celestial equator in half.

At the geographic pole, the celestial pole coincides with the zenith and the celestial equator coincides with the horizon.

On June 21 the declination of the Sun is $+23.5^o$. At the north geographic pole the Sun circles parallel to the horizon at an altitude of 23.5^o and does not set, resulting in 24 hours of continuous daylight. On December 21 the Sun's declination is -23.5^o and the Sun is south of the celestial equator. At the north geographic pole on this date, the Sun moves around the observer in a circle parallel to the horizon but 23.5^o below the horizon and does not rise.

In winter at mid-latitudes, the Sun rises south of east and sets south of west and the daytime is shorter than nighttime. In summer, when the Sun is north of the celestial equator, it rises north of east and sets north of west and there are more hours of daylight than darkness

At the north geographic pole, between March 21 and September 21, the Sun is above the horizon for 24 hours each day giving 6 months of continuous daylight. Between September 21 and March 21 the Sun's declination is negative and the Sun is below the horizon for 24 hours each day giving 6 months of continuous darkness. At the north geographic pole all objects with negative declinations remain below the horizon. Objects with positive declinations are circumpolar and have their diurnal circles entirely above the horizon. The reverse is true for the south geographic pole. In short, half of the sky is always visible and the other half is never visible at either geographic pole.

At the geographic equator, the celestial equator passes through the zenith and makes an angle of 90° with the horizon. The north and south celestial poles lie on the horizon. As seen from here, all objects rise and set vertically and there are no circumpolar objects. Every object is above the horizon for twelve hours and below it for twelve hours. The length of the daylight period is always twelve hours. The equator is also the only place where the Sun is at the zenith at noon on the dates of the equinoxes. The Sun can never be seen to pass overhead, through the zenith, at any latitude greater than 23.5^o, in either hemisphere. The parallel of latitude 23.5^o N. is sometimes called the Tropic of Cancer and its counterpart in the Southern Hemisphere is known as the Tropic of Capricorn.

III. Tutorial

Logon to **Skylab2** and select program No. 4, ***Celestial Sphere***. From the Celestial Sphere menu, select ***Animate Sun***. The computer will now draw an external view of the celestial sphere as seen from the latitude of TCNJ for the present date and time and then begin to animate both the

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diurnal and annual motion of the Sun. The diurnal motion can be seen as the motion of the Sun along its diurnal circle for whatever declination the Sun has today. The little cross on the celestial equator is the vernal equinox and one can simultaneously see the diurnal circle of this important point in the sky. The annual motion of the Sun shows up as the slow change in the Sun's declination from one day to the next and hence, a slight change in the Sun's diurnal circle from one day to the next. Pressing the space bar will stop the animation and a column of data will appear along the left margin of the screen. Do this. Among these data are the values for the right ascension and declination of the Sun and also its altitude and azimuth. Several different time readings will also appear but just pay attention to the **local apparent solar time (LAT)**. **The local apparent solar time is defined to be the hour angle of the Sun plus 12 hours.** This time may differ by several minutes from the time on our clocks or **zone time (ZT)**. The reason for this difference is explained in Exercise 8.

Now press the space bar again to restart the motion. Allow the animation to run through the year and watch the changing declination of the Sun result in a different diurnal circle from one day to the next. You can speed up this motion by changing the step size to one hour. Press the space bar occasionally to get a readout of the right ascension (RA) and declination (DEC) of the Sun. Pay attention to the changing places on the horizon where the Sun rises and sets as the seasons advance. Also note that the upper and lower transit points for the Sun change.

Now do the same thing for latitude 60° and also 20° in order to study the differences in the Sun's motion for different places on the Earth. You might try to experiment with other latitudes such as the equator and the pole.

Now press the Esc key until you are back at the Main Menu. From the Main Menu select the **Skymation** routine to investigation the motion of the Sun as seen on a rectangular chart of the sky. When the chart appears you will see a bunch of letters moving around on the chart. These letters represent the planets and the Sun. To identify which is which, open the Symbol window by pressing the letter S on the keyboard. Hit the "Esc" key to remove the window.

The animation you see is the orbital motion of the planets, including the Earth's. The Earth's motion, of course, shows up as the apparent motion of the Sun along the ecliptic. The Sun is identified as the object **H**, which stands for Helios, the ancient Greek God of the Sun.

NOTE WELL: The position of the Earth cannot be shown on this chart, for that would be a contradiction. You must realize that your eyes are on the Earth, off the monitor screen. That is, the chart on the screen is a compressed view of the entire sky or celestial sphere around you and the Earth. It may help to try and associate what you see on the computer screen with what you see in the planetarium.

Now find the wavy red curve. This is the ecliptic, the annual path of the Sun around the celestial sphere. The points where this curve crosses the celestial equator are the equinoxes.
The vernal equinox is at right ascension RA = 0:00 and the autumnal equinox is at RA = 12:00.

The default step is one day. That is, the sky is being represented for the same time on successive days. If you want things to go faster or slower, change the step size as usual. Allow things to run and concentrate your attention on the Sun. Pay attention to the changing right ascension (RA) and declination (DEC) of the Sun. Remember it is the changing declination of the Sun that causes its diurnal circle to change. If you want the stars shown also, you may obtain them from the options window. Find out in what constellation the Sun is located on your birthday. Is it the one you usually associate with your horoscope?

It is important to realize that the motions animated on the rectangular chart are what one sees from an Earth that is just a point in space. Therefore, what is seen on this chart is true for any observer and is independent of geographic position.

Now pay attention to the moving blue vertical line. This is your local celestial meridian (LCM). The motion of this line represents the rotation of the Earth. That is, for the same solar time from one day to the next, the position of the stars relative to your celestial meridian (LCM) changes by 1° or 4$^\mathrm{m}$ of time. We shall investigate this matter in another exercise.

IV. Assignment

Now return to the program ***Celestial Sphere*** and then select ***Animate Sun***. Your first objective will be to determine the date and time the Sun arrives at each of the equinoxes and solstices for the current year. Start with the vernal equinox and work your way through the year. Remember that the Sun may arrive at the equinoxes and solstices anywhere between the 19th and 23rd of the month, so you will have to search over these intervals to find the exact date and time for these events. Use the values for the RA of the Sun listed in the table in section **I** as the criteria for determining when the Sun is at one of the desired points. The Sun can arrive at any of these points at any time of the day as long as the RA is exactly the value listed in the table. Do not use the declination as the criterion or you will get the wrong answers. Make sure you work to the same precision as in Exercise 5.0 and record your data in the same way on the answer pages.

As you find the dates for the Sun to arrive at the equinoxes and solstices, stay on each date and determine the local apparent time (LAT) the Sun rises, makes UT, sets, and makes LT. To do this, proceed exactly the same way you did in Exercise 5.0 and use the same criteria for determining when the Sun rises, etc. When recording angles such as altitude, do not use colons. Instead use the symbols for degrees and arcminutes. Colons are to be used only for recording time.

After you have all the above data in hand for latitude 40° N., select one of the other latitude options below and repeat the same determinations. However, it is not necessary to determine the dates and times for the Sun to arrive at the equinoxes and solstices, since they should be the same. You need only copy the dates and times you found for latitude 40° N. onto the answer page for the other latitude.

The last part of this exercise is to draw the diurnal circles of the Sun for the above dates on the blank celestial sphere diagrams provided, one for each latitude. The data you compiled in the tables on the two answer pages will help you draw these circles. Also, some of the data will be used to annotate the diurnal circles that you draw.

Now proceed to do the above by following the steps listed below:

1. Now type "D" to call down the DATE menu, set your start date to March 19, and press Enter.
2. Type "O" for the options window menu and set your step size to 4 minutes. Press Enter.
3. Set the delay interval to 0.3 seconds and press Enter
4. Allow the animation to run and every now and then press the space bar to stop the motion and note the right ascension of the Sun.

NOTE: Use the values for the RA of the Sun as the criteria for these events and not the declination. When the right ascension (RA) of the Sun is exactly $0^h\ 00^m\ 00^s$, the Sun is at the vernal equinox. Try to stop the motion at this exact time, but no more than 1^s in right ascension (RA) different from this. You will find that there is an indeterminacy of about plus or minus 7 minutes in the local apparent time (LAT) to accomplish this. **Note: 24:00 = 0:00.**

5. Record the date and the local apparent time (LAT) at which the event occurs on the worksheet.
6. Now that you know the date, run the diurnal motion of the Sun backwards or forwards to determine the hour angle, altitude, azimuth, and local apparent time (LAT) when the Sun rises, makes upper transit (UT), sets, and makes lower transit (LT) on that date. **Warning: Do not run the diurnal motion backwards and forwards past midnight when trying to find lower transit (LT) or the computer will get the date mixed up.** If this happens, you will have to reset the date and start over.
7. Record these values on the answer sheet, using the degree and arcminute symbols when writing angles, such as, 25° 15'. **Do not use colons.**

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8. Repeat for the summer solstice, the autumnal equinox, and the winter solstice, using the right ascension (RA) as your criterion. You should know what the right ascension of the Sun has to be for these events. (See the table in section **I**.)
9. Repeat the above for the vernal equinoxes and solstices at the same longitude, but at the other latitude assigned by your instructor. Record the results on the other answer page.
10. Use a different blank celestial sphere diagram to draw the diurnal circles of the Sun for these dates at each of the two latitudes, as outlined in the next section. After you have drawn the diurnal circles, write the values of the local apparent times the Sun rises, makes upper transit (UT), sets, and makes lower transit (LT) next to points where the Sun does these things

V. How to Draw a Diurnal Circle

1. First use a protractor and locate the positions of the celestial poles. The latter are to be placed on the local celestial meridian (LCM) using the latitude angle. Don't know what to do? Refer to Exercise 6, where the concepts for this were presented.
2. Now draw a line connecting the poles. This line should pass through the center of the celestial sphere. This is the axis of rotation of the celestial sphere. Label it as such.
3. Now use the protractor and the latitude angle to find the two points where the celestial equator intersects the celestial meridian (LCM). Again, refer to Exercise 6 if you have forgotten the connection. Label the point on the upper LCM as **Σ** and the point on the lower LCM as **Σ′**.

   Warning: If your protractor is larger than the drawing of the celestial sphere, you will need to make marks that are off the **LCM** to indicate any angle you measure. To find where these marks should be on the **LCM**, lay a ruler along your mark and the center of the celestial sphere, then make a mark where the ruler crosses the **LCM**. This mark then indicates the actual point of upper or lower transit.

4. Now draw the celestial equator as an ellipse passing through these two points and the **E**ast and **W**est points. Make the half of the ellipse on the back of the celestial sphere to be a dashed curve. This is the diurnal circle of the Sun on the days of the equinoxes.
5. Indicate somewhere along the celestial equator that the declination is $0°$ and draw 2 short, bold arrows along the circle, one on the back and one on the front, indicating the direction of diurnal motion.
6. At this point, check your diagram with those from Exercise 6. They should look consistent.
7. Look up the declination of the Sun when it as at the summer solstice. Then use a protractor to find the points of upper transit (UT) and lower transit (LT) for the Sun on the **local celestial meridian** by measuring an angular distance from the celestial equator equal to the declination. Label these points as **SUT** and **SLT**.

   Remember to set the protractor to measure declination from the celestial equator and not the horizon.

8. Once the points of upper and lower transit have been properly located on the **LCM,** connect these two points with a faint pencil line.
9. Through the point where the above line intersects the north-south (**N-S**) line, draw another faint guideline parallel to the east-west (**E-W**) line from horizon to horizon. The two points where this last line touches the horizon are the points where the Sun rises and sets. Label them as **SSR** and **SSS**.

10. Now draw the diurnal circle of the Sun as an ellipse that passes through the upper transit point (SUT), lower transit point (SLT), and the rise and set points. Again make the back half of the curve a dashed line.
11. Indicate the declination of the Sun along this circle and draw 2 short, bold arrows along the circles, one on the back and one on the front, indicating the direction of diurnal motion.
12. Repeat the above to draw the diurnal circle of the Sun for the winter solstice, except now use the declination of the Sun when it is at the winder solstice. The angle is negative, so you are to measure south from the celestial equator to find the upper and lower transit points. Replace **S** with **W** for labeling the UT, LT, Rise, and Set points.
13. When the diurnal circles are completed, write the values of local apparent time (**LAT**) when the Sun rises, makes upper transit, etc., next to the corresponding points along the diurnal circle.
14. Repeat all of the above on the other blank celestial sphere diagram for the other latitude for which you collected data.

When you have completed drawing and annotating the diurnal circles of the Sun for the different latitudes, study these diagrams to see how different the path of the Sun in the sky is from season to season and latitude to latitude. Note also the difference in times when the Sun rises and sets. Relate the fraction of the diurnal circle above and below the horizon to the length of daylight and nighttime and how this varies from latitude to latitude**.**

CHANGING DIURNAL CIRCLES OF THE SUN
WORKSHEET

Latitude_____

Date of VE: _____ , Local Apparent Time (LAT): _____

Event	Hour Angle	Altitude	Azimuth	LAT
Sunrise				
UT				
Sunset				
LT				

Date of SS: _____. Local Apparent Time (LAT): _____

Event	Hour Angle	Altitude	Azimuth	LAT
Sunrise				
UT				
Sunset				
LT				

Date of AE: _____, Local Apparent Time (LAT): _____

Event	Hour Angle	Altitude	Azimuth	LAT
Sunrise				
UT				
Sunset				
LT				

Date of WS: _____, Local Apparent Time (LAT): _____

Event	Hour Angle	Altitude	Azimuth	LAT
Sunrise				
UT				
Sunset				
LT				

Diurnal Circles of the Sun
Answer Diagram

Latitude _____