Exercise 5.0

LUNAR MOTION, ELONGATION, AND PHASES

I. Introduction

The Moon's revolution in orbit around the center of gravity (barycenter) of the Earth-Moon System results in an apparent motion of the Moon in the sky eastward relative to the Sun and, therefore, the **phases** of the Moon. When measured with respect to the fixed stars, the Moon's period of revolution is 27.3 days. That is, this is the length of time it takes the moon to revolve 360^o in its orbit and is called the **Sidereal Month**. Hence, **the Moon's rate of motion is 13.20 degrees per day, eastward with respect to the fixed stars**. However, **the Moon's rate of motion is 12.20 degrees per day eastward with respect to the Sun**. This is because the Sun has an apparent motion eastward, along the ecliptic, of 1^o per day with respect to the fixed stars. Recall that this apparent motion of the Sun is a reflection of the Earth's orbital motion around the Sun. The time it takes the moon to appear to move around the celestial sphere and return to the Sun is called the **Synodic Month**. This takes 29.5 days and is also the length of time it takes the moon to go through a complete cycle of its phases.

The Moon's orbital motion around the Earth causes the angle between the Sun and the Moon, as seen in the sky from the Earth, to continuously change. This angle is called **Elongation.**

Elongation is the angular distance of an object measured along the ecliptic, eastward or westward from the Sun.

When the Moon has a specific elongation, it has a specific phase, and rises and sets at a specific time. The table below lists the elongations of the Moon, the corresponding aspects or configuration names, and phases of the Moon.

Elongation	Aspect/Configuration	Phase
0°	Conjunction	New Moon
1 - 89° E		Waxing Crescent
90°E	Eastern Quadrature	First Quarter
91 - 179° E		Waxing Gibbous
180°	Opposition	Full Moon
179 - 91° W		Waning Gibbous
90°W	Western Quadrature	Third/Last Quarter
89 - 1°W		Waning Crescent

The approximate time the Moon rises for a given elongation can be calculated from the following equation:

Time of lunar event = Time for corresponding solar event - Elongation in time units.

This may be written in symbolic form as: $\mathbf{T_M = T_{\odot} - T_E}$

For example, if we wanted to calculate what time the Moon will rise when it has a certain elongation, the corresponding time for the Sun would be sunrise. For sunrise use 6:00 and for sunset use 18:00. You should know what times to use for UT and LT of the Sun. For example, the full Moon, which has an elongation of 180^o or 12^h, rises at sunset and sets at sunrise.

There can be more than an hour error in this calculation because the equation assumes that both the Sun and Moon are always on the celestial equator. Nevertheless, the above equation yields a good approximation, especially when the Sun and Moon are near the CE. The elongation angle, when expressed in time units, is simply the amount of time that the Moon lags the Sun, when the elongation is measured eastward, or precedes the Sun, when measured westward. In this exercise, we explore these relationships.

II. Tutorial

Logon to SKYLAB2 and select program SKYMATION. A rectangular chart will be drawn that represents the entire celestial sphere. The ecliptic appears as a wavy red curve. As soon as the chart is displayed, an animation will begin that shows the changing positions of the Sun, Moon, and planets in one day steps for the equatorial frame of reference. The positions of each of the planets, the Sun, and the Moon are indicated by small circles that are letter coded. The key to this code can be viewed as a window by typing "S" for symbols.

Open the Options window and select "Plot Stars." Now watch the animation run for a while to see what is happening and how the moon changes its position from day to day relative to the stars and the Sun. Note that the moon moves about 13 degrees eastward each day relative to the stars. Since the Sun moves eastward by 1 degree per day, the moon gains about 12 degrees per day on the Sun.

Recall that elongation is the angular distance of an object from the Sun as measured along the ecliptic. Watch the elongation of the moon changing. Occasionally press the space bar to stop the motion and see if you can estimate the Moon's elongation angle and direction. To do this, remember that each hour of RA is 15^o.

Now change the step size to 1 hour and delay factor to 0.5 second. This will enable you to see details happening over each day. Now allow the animation to begin. Note the planets hardly move at all in 1 hour, but the Moon's motion is noticeable.

The large jumps in the position of the LCM are the result of seeing where it is relative to the stars every hour and thereby indicating the sidereal time for the corresponding ZT indicated at the top of the screen. This eastward motion of the LCM is an alternative way of representing diurnal motion of the celestial sphere westward relative to the LCM. Recall that the parallels of declination are the diurnal circles.

Now we shall animate the Moon's daily motion in another way. From the main menu of SKYLAB2, select Lunamation. Then from the Lunamation Menu, select NO. 2, Diurnal Motion. Now a diagram will appear on the screen with three different panels. The center panel represents an observer-centered diagram of the horizon system, as seen projected into the plane of the prime vertical. The diurnal circles of the Sun and Moon are shown to be one and the same. The horizontal line with east on the left and west on the right is meant to represent the east-west line in the plane of the celestial horizon. The vertical line represents the plane of the LCM with upper transit at the top and lower transit at the bottom. The observer is located at the center of the diurnal circle.

The animation depicts the diurnal motion of the Sun and Moon around the observer from east to west. This is the result of the rotation of the celestial sphere (or Earth). At the same time, the orbital motion of the moon around the Earth shows up as a slow eastward displacement of the moon relative to the Sun at the rate of 12.2 degrees per day (13.2 degrees per day with respect to the fixed stars). The angle between the Sun and the LCM indicates the local apparent solar time, LAT. The angle between the Sun and the moon, as measured along the circle of motion, is the Moon's elongation. This angle is related to the phase of the moon.

The lower left panel shows a view of the phase of the Moon as we would see it in the sky. The panel on the lower right is a view of the Moon's position in its orbit relative to the Earth; the direction of the Sun is to the right. This view is from a point in space far above the plane of the Moon's orbit.

At the top right of the screen are given the date and the ZT. Watch the animation continue through one lunar cycle or more, until you understand these motions. The animation can be stopped and started by pressing the space bar. Try to run the animation at a speed where you can stop the animation just as the moon rises or sets. Note that the time the moon rises or sets depends on the elongation or phase of the moon. Both of these events are retarded by about 48.8 minutes from the previous day (12.2 deg. x 4 min/deg.). When you have gained experience running the lunamation program, you are ready to advance to Part **III**.

III. Assignment

A. Determining the Lunar Orbital period

Run the program Skymation with the step size set to 3 hours and a delay time of 0.6 seconds. Plot the stars and a grid. Now set Skylab to trail the Moon's motion (under Options) and then quickly press the space bar to halt the animation. Now get a printout (#1)

On the answer sheet, write down the day and time (ZT) as answer No. 1. Also take careful note of the position of the Moon with respect to the stars and record the Moon's RA in hours and minutes and DEC. in degrees.

Now start the animation and allow the Moon to complete exactly one orbit relative to the fixed stars and then press the space bar to stop the motion when the Moon returns to the position recorded above. Do not allow the moon to move past this point. You may have to practice before you are able to do this! Record the date and time when this happens on the answer sheet as No. 2 and get a hardcopy of the screen. The two dates that you have recorded will be analyzed later.

B. Determining Moonrise and Moonset

Now we shall investigate the diurnal and orbital motions of the Moon and the resulting changes in the Moon's elongation and rising and setting. To do this, run the Diurnal Motion program under Lunamation. Set the step size to 4 min. and the delay factor to 0.80 seconds. In this way you should be able to determine the time of moonrise or moonset with a precision of 4 minutes. Now run the program and stop the animation when the center of the moon is just rising or setting (at the East or West Point) for the date assigned to you. Get a printout of this screen (#3).

Now we shall watch the diurnal motion of the moon and planets in the horizon system. Exit Lunamation and activate the Skymation program. When the animation begins change the date to the one that is a few days earlier than the one you have been using above. Set the step size to 4 min. and the delay factor to 0.80 sec. After you return to the animation again, type "C" to open the coordinates menu, then select "Horizon" and press Enter. Now pan up by 15^o, and then zoom out by a factor of 3. This will provide a more panoramic view of the horizon system.

Allow the animation to run and watch the planets move along their diurnal circles relative to the horizon and LCM. The latter is the red vertical line fixed at azimuth 180. Recall that the east point of the horizon is at azimuth 90 and west is at 270. Notice that the planets rise from the east, ascend to upper transit on the LCM, and then descend to set in the west. This motion represents their diurnal circles in the horizon system. Watch the animation for a few days or until you understand what is taking place.

Remain in the horizon system and under the Options menu, select to trail the Moon. **Do not request a grid or plot the stars, otherwise the Moon will not trail.** Allow the animation to continue and watch the changing diurnal circle of the moon from one day to the next. This results from the fact that the orbit of the Moon is inclined to the plane of the celestial equator and its declination is changing. Now reset the date to the one assigned and try to stop the animation when the Moon either rises or sets (altitude=0^o). Obtain a hardcopy of the screen (#4) and write the ZT on the answer sheet as No. 11. Does this value agree with the ZT value at the top of printout #3? If not, you should be able to explain why not. Hint: consider the declination of the Moon and Sun and the difference between ZT and LAT. Answer No. 11 is the more correct answer.

Proceed to analyze the charts you have obtained:

No. 3. Calculate the number of days and hours between the two dates that you recorded for answers Nos. 1 and 2. Convert the hours to a decimal part of a day. Then record the difference in days and decimal parts thereof as answer No. 3 on the answer sheet. Show your calculation at the bottom of the answer page.

No. 4. The above answer is the time it takes the Moon to complete one orbit. What is this period of the Moon's motion called? To answer, think about how the position of the Moon was determined and compare with the definitions of the two kinds of lunar orbital periods given in the introduction.

No. 5. Now take hardcopy #3 and draw a line from the observer to the center of the Sun and another line from the observer to the center of the Moon. The angle between these two lines is the elongation of the Moon. Measure this angle with your protractor and record along with the direction.

Draw and arc from the Sun to the Moon in a way similar to that shown in the sample chart at the end of this exercise. Draw an arrowhead at the Moon and print the elongation of the moon along this arc as shown in the sample diagram. Be sure to indicate whether the elongation is E/W. The value of the elongation of the moon that you have measured should agree with the phase of the Moon given in the lower left panel of the page. If they do not agree, you are doing something incorrectly.

No. 6. Now take the value you measured for the elongation of the Moon and covert the degrees to hours and minutes of time using the angular rate of rotation of the Earth. Prefix the proper algebraic sign depending on whether the elongation is east or west of the Sun.

No. 7. Use the above value and the equation in section **I** to calculate the LAT at which the Moon rose or set. Show the steps of your calculation in the upper left hand corner of chart #3 exactly as shown on the sample chart. Also record your answer on the answer page.

No. 8. On chart #3, draw an arc from the point of upper transit to the Sun to represent the hour angle of the Sun. Place an arrowhead at the Sun to indicate the sense of direction. Use you protractor to measure this angle and record this as No. 8 on the answer page with the correct direction. Also label and write the value of the hour angle along the arc exactly as shown in the sample.

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No. 9. Convert the above hour angle of the Sun to hours and minutes of time using the angular rate of rotation of the Earth, as you did for the Moon’s elongation. Record this on the answer page.

NO. 10. Now use this value and the equation:

$$\mathbf{LAT = HA_{\odot} + 12{:}00}$$

to compute the LAT for the Moon to rise or set another way. Show this calculation in the top, right corner of chart #3, following each step exactly as shown on the sample chart. Also record your answer on the answer page.

The answers to No. 7 and No. 10 should agree within 4 minutes. Compare these results with the value of the ZT displayed at the top right of the chart. The difference between the ZT and your two values may differ by up to 90 minutes or so. Part of this difference is the equation of time (Exercise 8.0). There are also contributions from the obliquity of the ecliptic, the inclination of the Moon's orbit, and the distortion of the Moon's diurnal circle on chart #3 introduced by the software and printer.

Also compare the answer in No. 11 with your two calculated values. They may not agree with this either because of the assumptions necessary for the above equations to be valid. No. 11 is probably the most accurate value.

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Lunar Motion Answer Sheet

1. Date to begin Moon's motion: Date ____________________; ZT__________________.

   Moon's position on printout 1: RA ____________________; DEC ________________.

2. Date of end of Moon's motion: Date____________________ ; ZT__________________.

3. Time for one lunar orbit in days and decimal parts thereof: ________________.

4. Name of Period (See Sec. **I**): ___________________________________.

5 Lunar Elongation in degrees as measured on printout #3: ______________________.

6. Above elongation of the Moon converted to hours and minutes: ______________________.

7. Calculated LAT of Moon rise/set using the equation of Sec. **I**: ______________________.

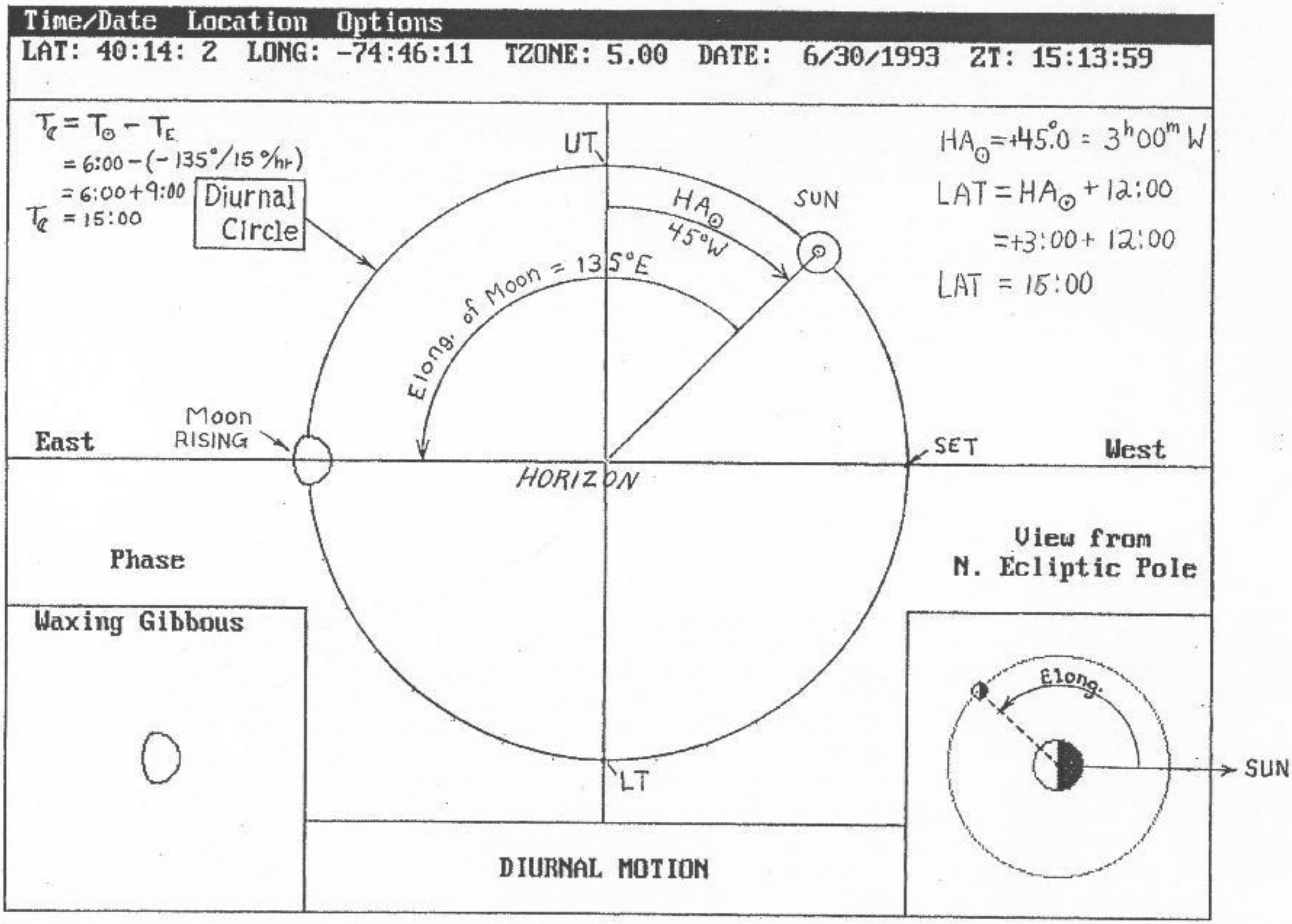
8. Hour angle of the Sun in degrees as measured on printout #3: ______________________.

9. Hour angle of the Sun converted to hours and minutes: _________________.

10. Computed Lat from above hour angle of Sun: _________________.

11. Moonrise/set from horizon system printout (#4): ________________.

Show here the details of the calculation for the answer to #3:



Time/Date Location Options
LAT: 40:14: 2 LONG: -74:46:11 TZONE: 5.00 DATE: 6/30/1993 ZT: 15:13:59
T☾ = T☉ − T_E
= 6:00 − (−135°/15°/hr)
= 6:00 + 9:00
T☾ = 15:00
Diurnal Circle
UT
HA☉ 45°W
SUN
Elong. of Moon = 135°E
HA☉ = +45°.0 = 3h00m W
LAT = HA☉ + 12:00
= +3:00 + 12:00
LAT = 15:00
East
Moon RISING
HORIZON
SET
West
Phase
View from N. Ecliptic Pole
Waxing Gibbous
Elong.
SUN
LT
DIURNAL MOTION