Resonant Ion Scattering and the Integrated Atomic Scattering Cross-section for Determining Ion densities in the Modeled Shock Fronts of EM Carinae and HD159176

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Abstract

The current literature is rather vague regarding how to calculate the exact numerical value of the resonant ion scattering cross section that should be used for a specific bandpass of finite width. Such a value was needed in order to calculate the ion density in the shock front of a close binary star system, based on a modeling of an ultraviolet wind-line profile, using *IUE* spectra. Therefore, we have carried out a numerical integration, in wavelength-space, of the exact expression for the cross section over two bandpasses of interest. The exact expression employed was that derived from a solution of the Abraham-Lorentz equation. The numerical results depend on the resonant wavelength, which is taken to be at the center of the bandpass. Most texts on the subject derive an expression for the scattering cross section in frequency-space, based on the assumption that the radiation reaction term in the Abraham-Lorentz equation may be approximated by a resistive term. The integral of this cross section over the entire spectrum is independent of the resonant frequency, except for the transition probability. This has limited practical use when dealing with fluxes measured in a bandpass of finite width expressed in wavelength units and scattering is the only mechanism for producing the observed fluxes. Such is the case when dealing with the low densities encountered in stellar winds and shock fronts. Integrated cross sections that depend on the resonant wavelength are used to determine the number density of C IV ions in the shock front found in EM Car and the C IV and N V densities for the shock in HD159176. These results indicate high ionization temperatures in these shocks, as expected.

1. Introduction

This study was prompted by an undertaking of one of us, RJP, to model the UV light curves for *IUE* bandpasses encompassing a wind-line profile for hot eclipsing binary stars. To accomplish this in part, it was necessary to calculate the amount of radiation that is scattered towards the observer, in a specified bandpass, by certain ions in a given volume element of a shock front formed by the colliding winds of the stars. In so doing, one needed to determine the product of the ion density and the atomic scattering cross-section. In turn, the number density of ions, such as C IV, in the shock could be found, if the atomic scattering cross-section were known. The problem then became to find the value of the scattering cross-section, $\sigma_{scat}(\Delta\lambda)$ for the appropriate bandpass, $\Delta\lambda$. One would think that the theory of atomic scattering that is given in texts would be clear on this matter, but this is not the case. Most authors develop the theory of atomic scattering in frequency space rather than wavelength space. This is because angular frequency, ω, is a natural parameter that appears in the harmonic oscillator equation. That is, ω is related to the ratio of the elastic restoring force constant to the mass of the electron, more specifically, $\omega^2 = k/m_e$. One purpose of this paper is to help clarify this matter when dealing with spectrophotometric data that employs wavelengths. The

other is to compute the ion densities in the shock fronts that have been previously modeled for EM Carinae (Pfeiffer, R. J. and Stickland, D. J. 2004) and HD159176 (Pfeiffer *et al*. 1997).

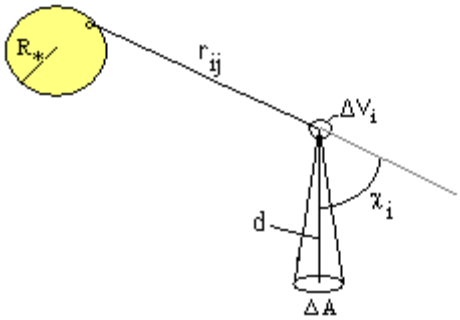
2. Resonant Ion Scattering from a Static Shock

Let $F_j(\lambda)$ be the unpolarized, monochromatic flux at wavelength λ, emanating from the jth element of surface area of the photosphere of a star and incident on an ion located in the shock front of an eclipsing binary star system. The total monochromatic power scattered in all directions by that ion is (Stone, J. M. 1963, p. 343):

$$P_{\text{scat}}(\lambda) = \sigma_{scat}(\lambda)F_j(\lambda). \tag{1}$$

An equivalent equation is (4-31) in Mihalas (1978). Actually, (1) defines $\sigma_{scat}(\lambda)$, the atomic scattering cross section at wavelength λ. Again, if one assumes that the incident beam is unpolarized, the power in bandpass $\Delta\lambda$ scattered by a volume element ΔV_i with ion density n, into solid angle $\Delta\omega$ through angle χ_i is (See Fig. 1):

$$\Delta P_i(\Delta\lambda) = \sigma_{scat}(\Delta\lambda)n\Delta V_i\left[\frac{3}{16\pi}(1+cos^2\chi_i)\right]F_{ij}(\Delta\lambda)\Delta\omega. \tag{2}$$



The factor in the square brackets is the normalized, angular distribution factor for scattering. The quantity $F_{ij}(\Delta\lambda)$ is the flux emanating from the jth surface element of a stellar photosphere and incident on the ith volume element of the shock. The volume element of the shock, ΔV_i, is located at a distance r_{ij} from surface element j of a star, which has radius R_*. Equation (2) is based on the development given by Stone (1963), who uses *I* (irradiance) for flux but otherwise presents a rather clear presentation of scattering theory not found in most sources. If ΔA is the area of a detector at distance d from the volume element ΔV_i, then $\Delta\omega = \Delta A/d^2$, and the flux falling on the detector is

$$\Delta F_i(\Delta\lambda,\ d) = \Delta P_i(\Delta\lambda)/\Delta A = \sigma_{scat}(\Delta\lambda)n\Delta V_i\left[\frac{3}{16\pi}(1+cos^2\chi_i)\right]F_{ij}(\Delta\lambda)/d^2. \tag{3}$$

To simplify the following discussion, let $\mathcal{S} = \sigma_{scat}(\Delta\lambda)n\Delta V_i\left[\frac{3}{16\pi}\left(1+cos^2\chi_i\right)\right]$. Now let $F_{*j}(\Delta\lambda)$ be the flux emanating from a surface element of the star. Then

$F_{ij}(\Delta\lambda) = F_{*j}(\Delta\lambda)\alpha(\Delta\lambda)R_*^2/r_{ij}^2$, where $\alpha(\Delta\lambda)$ is an attenuation factor caused by the intervening wind envelopes. Substituting the latter into (3) yields:

$$\Delta F_i(\Delta\lambda,\ d) = \mathcal{S}_j \frac{F_{*j}(\Delta\lambda)\alpha(\Delta\lambda)}{d^2}\frac{R_*^2}{r_{ij}^2}. \quad (4)$$

Now the total observed stellar flux from the system, $F_*(\Delta\lambda,\ d) = F_*(\Delta\lambda,\ R_*)R_*^2/d^2$, is known from the *IUE* data. Here, the quantity $F_*(\Delta\lambda,\ R_*)$ is the flux emanating from the photosphere of a star in the bandpass $\Delta\lambda$, before it is attenuated by the wind envelope by an amount $\alpha(\Delta\lambda)$. Therefore, the quantity $\mathcal{S}_j F_{*j}(\Delta\lambda)\alpha(\Delta\lambda)R_*^2/d^2$ in (4) may be replaced by $\mathcal{S}_j F_*(\Delta\lambda, d)\mathrm{P_j}\alpha(\Delta\lambda)$, where $\mathrm{P_j}$ is the value of a partitioning function for a given surface element of the star. Hence, (4) becomes

$$\Delta F_i(\Delta\lambda,\ d) = \boldsymbol{S_i}\, F_*(\Delta\lambda,\ d)P_j\alpha(\Delta\lambda)\frac{1}{r_{ij}^2}, \quad (5)$$

This essentially is the derivation of equation (3) in Pfeiffer *et al.* (2004), but with a slightly different notation and considering only one of the stars in the binary. However, this derivation is not presented there. Furthermore $F_*(\Delta\lambda, d)\alpha(\Delta\lambda)P_j$ is the equivalent of the parameter F_{ijk} in equation (3) of the above reference, where *k* indexes a specific star in the binary.

The problem then becomes to properly partition the total observed bandpass flux among the different components of the system, which not only includes the photospheres of the stars but also the wind envelopes. For the photospheres, P_j includes limb darkening, polar brightening (also called gravity darkening when referring to equatorial latitudes), and Lambert's law.

To find the total observed bandpass flux scattered by the ions in the shock, one numerically integrates (5) over the volume of the shock, which is assumed to be static. This integration is itself a very complex procedure, since χ_i varies from one volume element to another. In practice, this angle is found from the inner vector product of the position vector of the volume element relative to an element of area of a photosphere and the unit directional vector of the observer. One must also determine $\alpha(\Delta\lambda)$ by taking into account the attenuation of the photospheric flux as it traverses the wind envelope before encountering the volume element in the shock and after it is scattered and traverses the wind or winds again in the direction of the observer. To accomplish this, one needs to model the opacity in the wind as a function of path length. In addition, one has to contend with two

stars and develop the partitioning function for the observed flux. However, these details are not our concern here but may be found in Pfeiffer and Stickland (2004). Suffice to say that once the integration over the volume of the shock front is done, the resulting flux from the shock is added to the fluxes emanating from the photospheres and winds of the stars, taking into account eclipsing effects on all parts of the system. The emission from the wind envelope of each star, in bandpass $\Delta\lambda$, is calculated using a program, *SEI*, developed by Lamers *et al.* (1986), which was designed to model a wind-line profile. This program has been previously adapted for use in binary systems by Pfeiffer *et al.* (1994). The total modeled flux must then be in agreement with the light curve observed at a given Keplerian orbital phase. The geometric parameters of the shock as well as all other parameters, such as $\sigma_{scat}(\Delta\lambda)n$ in (5), are varied until such a fit is achieved. When the scattered flux contributed by the shock to the total systemic flux is correctly modeled, the number density of the ions in the shock may be found, provided the value of $\sigma_{scat}(\Delta\lambda)$ is known. This is the matter we now address.

3. The Integrated Scattering Cross-Section

The total or integrated scattering cross-section, σ_I, as presented in some texts, is obtained by integrating the scattering coefficient over all frequencies from zero to infinity, *viz*.,

$$\sigma_I = \sigma_T \int_0^\infty \frac{\omega^4 d\omega}{(\omega^2 - \omega_0^2)^2 + (\gamma\omega^3/\omega_o^2)^2}, \qquad (6)$$

where $\sigma_T = \frac{8\pi e^4}{3m_e{}^2c^4}$ is the Thomson cross section, $\omega = 2\pi\nu = 2\pi c/\lambda$, e is the electrical charge on the electron, m_e is the mass of the electron, c is the speed of light, $\gamma = 2e^2\omega_o{}^2/(3m_ec^3)$ is the classical radiative decay constant, and ω_o is the resonant frequency of an atomic transition. The above expression is equation (14-15) in Stone (1963). Similar expressions are equations (4-81) in Aller (1963), (17.63) in Jackson (1974), (4-34) in Mihalas (1978), (6.71) in Padmanabhan (2000), and (3.62) in Rybicki and Lightman (1979). Some authors use the symbol Γ instead of γ. One would suppose that these authors all give the same expression, but this is not the case. The disagreement concerns the 2nd term in the denominator of the integrand in (6). The expression for this term depends on what assumptions are made in the solution of the Abraham-Lorentz equation. Let us define this term to be u. Following Jackson, and Rybicki and Lightman, we introduce the characteristic time, $\tau = 2e^2/(3m_ec^3)$, so that γ or $\Gamma = \tau\omega_o{}^2$. Then for Mihalas, u is $\gamma^2\omega^2$ or $u = \tau^2\omega_o{}^4\omega^2$. Rybicki and Lightman give $u = \tau^2\omega_o{}^6$, which we suspect is a misprint, and should have

been $u = \tau^2\omega^6$. Stone gives $u = (\gamma\omega^3/\omega_0^2)^2$ or $\tau^2\omega^6$, and Padmanabhan gives $u = \tau^2\omega_o{}^4\omega^2$ in agreement with Mihalas. Our interpretation of Jackson's equation (17.63) is that it is in agreement with the expression given by Stone. This solution is obtained by taking the resistive term in the Abraham-Lorentz equation to be zero and keeping the radiation reaction term. That is, there is no energy loss by the harmonic oscillator to other effects (such as atomic collisions) other than quantum radiation (radiative scattering) and that one is dealing with a steady state solution. Hence, $\Gamma' = 0$ in $\Gamma_t = \Gamma' + \Gamma$ (Jackson, equation 17.61). Stone and Rybicki and Lightman do not approximate the radiation reaction term by using a resistive term, but Aller, Mihalas, and Padmanabhan do. This leads to the differences in the solutions among the above authors.

Of course, quantum mechanics introduces the oscillator strength or transition probability, f_{ij}, so that

$$\sigma_{scat} = \sigma_I f_{ij} . \tag{7}$$

For the following discussion, only the classical case shall be considered, since the final results need only to be multiplied by the transition probability.

The most transparent derivation of (6) is that given by Stone (Chapters 12 and 14) and the most rigorous by Jackson (Chapter 17). Hence, (6) is the correct and most rigorous solution when one is dealing with measured fluxes that are the result of just scattering by an ion and not scattering plus absorption (total extinction, as defined by most authors). Aller's exposition on this matter which leads to his equation (4-81) is somewhat confusing. On page 169, he states that the absorptivity, α_ν, as given by equation (4-73) was derived from a consideration of light scattering. In reality, the derivation of (4-73) was carried out on the basis of pure absorption using a resistive term in the equation of harmonic motion, (4-61), rather than a radiation reaction term.

The usual assumption made by some authors is that the profile of the scattering coefficient is so sharply peaked and the term *u* so very small, its exact value is not important. The replacement of the radiation reaction term by only a resistive term appears to be done in order to achieve and expression for σ that may be readily integrated. As we show, this turns out to be numerically valid for adequately wide bandpasses. Since the advent of high speed computers, it appears that no one has attempted a numerical integration of (6) for finite bandpasses.

However, even (6) is somewhat heuristic and perhaps misleading, for it is not the fundamental expression that defines the classical scattering cross-section, σ_{scat}. The latter only has meaning through the expression (1). The atomic cross section for scattering has no independent meaning outside of equation (1). That is, $P_{scat}(\Delta\lambda)$, or $P_{scat}(\Delta\omega)$, has a value that is specific for a particular bandpass and is to be found only by convolving the expression

$$\sigma(\omega) = \sigma_T \left[\frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + \left(\gamma\omega^3/\omega_0^2\right)^2} \right] \tag{8}$$

with the flux $F(\omega)$ when integrating over the bandpass of interest. In other words, when carrying out the integration indicated in (6), it is usually assumed that the radiation field is constant over all frequencies or wavelengths, which is never the case. Aller (1953, page 166) is the only author who warns about this, saying that F_ν should not vary appreciably with frequency near the resonant frequency. Additionally, most authors assume that σ is such a sharply peaked function, the contribution to the integration over frequency is negligible except in the region immediately near the resonant wavelength or frequency, even when one integrates to infinity. We decided to thoroughly investigate this assumption and find out how wide the bandpass must be for this to be true, using the exact expression, equation (8), for the cross section in the case of pure scattering.

Even if (6) is assumed valid, independent of $F(\omega)$, the integral has no analytical solution. Hence the need to use the alternative version of (6) that is derived using the resistive term in the Abraham-Lorentz equation in place of the radiation reaction term. In addition, it is assumed that $\omega \approx \omega_o$, and ω^2-ω_o^2 may be approximated by $2\omega(\omega-\omega_o)$ (Aller, p. 164; Mihalas, p. 83; Padmanabhan, p. 267; Rybicki and Lightman, p. 101). With these simplifying assumptions, and transforming variables from ω to ν, the integration leads to

$$\sigma_I = \frac{\pi e^2}{m_e c}, \tag{9}$$

when integrating from 0 to $+\infty$. There is an additional factor of 2π when integrating using ω as the variable of integration (Jackson, equation 17-73; Rybicki and Lightman, equation 3-65a). It is interesting to note that (9) is independent of the resonant wavelength or frequency. Jackson (page 805) states that (9) is obtained by neglecting the radiation reaction term and this is equivalent to assuming the width Γ_t is independent of the frequency. In other words, the decay time is assumed to be the same for all transitions. This is clearly seen in the evaluation of the integral as given by Aller (1963, p.166f). This implies that it is the integration of (8) over an infinite bandpass, with the above

assumptions to simplify the integral, that causes the loss of dependency of the cross section on the resonant frequency. Our investigation, discussed below, has shown that this is not exactly correct.

Of course, (9) is of little use unless it is used in conjunction with a broadening function that introduces the resonant frequency when calculating line profiles. The most straightforward exposition of this is given by Swihart (1963, p. 131f.) and also by Lamers *et al*. (1999, p. 198f.). However, as we show, for all practical bandpasses involving pure scattering, σ is dependent on the resonant wavelength λ_o, independently of a broadening function, but strangely enough, σ is independent of the resonant frequency.

4. The Bandpass Dependent Value of σ_{scat}

We decided to investigate how good the above simplifying assumptions to obtain (9) are by numerically integrating the exact expression, equation (6), for several bandpasses of astrophysical interest. Additionally, this was done using wavelength units in centimeters instead of frequency. Therefore, to make a comparison, we needed to convert the usually quoted numerical value for σ_I, 2.65 x 10^{-2} cm^2-Hz, to units of cm^2-cm. Such a conversion may be computed from

$$\sigma = \sigma_I \lambda_o^2 f / c, \tag{10}$$

The latter is given by Aller (1963, p. 302), when using wavelength units for computing the equivalent widths of broadened lines. Also see the discussion by Unsöld *et al*. (2001, p. 203f.) regarding line broadening. It comes about by changing the variable of integration from dν to dλ, since $(\lambda^2/c)\, d\nu = |d\lambda|$ and λ is evaluated at λ_o. We prove the validity of this below. Values for the constants in (9), with precision to at least 7 places, were taken from Cox (2000). So a more precise value for σ_I is 2.65376 x 10^{-2} cm^2-Hz, with uncertainty in the last place. Now, for one of the C IV resonant wavelengths taken from Striganov and Sventitskii (1968), equation (10) gives σ_I = 2.12172 x 10^{-22} cm^2-cm, (uncertainty in the last place) with *f* taken to be unity. We shall compare the results from our numerical integration with this value.

Converting (8) to wavelength units and then integrating over a finite bandpass centered on the resonant wavelength we get:

$$\sigma_{\Delta\lambda} = \sigma_T \int_{\lambda_i}^{\lambda_f} \frac{d\lambda}{(1-\frac{\lambda^2}{\lambda_o^2})^2 + \eta\frac{1}{\lambda^2}} \tag{11}$$

In (11), η is the constant $4\pi^2c^2\tau^2 = 16\pi^2e^4/9m_e^2c^4$, which has the value 1.39303 x 10^{-24} cm^2. Notice that the denominator in (11) is dimensionless and that the dimension of the integral is just cm, whereas the constant outside the integral (the Thompson cross-section) has units of cm^2. Again, values for the constants were taken from Cox (2000).

Values for $\sigma_{\Delta\lambda}$ were calculated by writing a *FORTRAN* program that numerical integrates (11) by the method of quadrature with unit weights on a SunBlade 150 work station. In order to avoid inconsistent results, σ must be evaluated at λ_o and multiplied by dλ in one of the steps. This was accomplished by starting the integration at λ_o and then integrating from there to shorter and longer wavelengths separately, since the function is not symmetrical about λ_o. Numerical values for the step in the integration, dλ, were used ranging from 1 x 10^{-8} cm to 1 x 10^{-16} cm. It was found that the integral converges to essential the same number, significant to 6 places, after dλ is reduced to 1.0 x 10^{-13} cm. However, the value for $\sigma_{\Delta\lambda}$ increases slightly with increasing bandwidth. The results for one of the resonant wavelengths for C IV, λ_o = 1548.185Å (Striganov *et al.*, 1968) are listed in Table 1. Again, uncertainty there is in the last decimal place. The last entry in Table 1 is the result using equation (10).

Table 1

Values of the integrated scattering cross-section centered on the resonant wavelength 1548.185Å for the C IV ion for several different bandpasses. The transitional probability is set equal to 1.0

Δλ(Å)	Bandpass (Å)	σ(cm^2-cm)
0.1	1548.135 – 1548.235	2.12012e-22
0.5	1547.935 – 1548.435	2.12140e-22
1.0	1547.685 – 1548.685	2.12156e-22
1.2	1547.558 – 1548.785	2.12158e-22
2.0	1547.185 – 1549.185	2.12164e-22
4.0	1546.185 – 1550.185	2.12168e-22
10.0	1543.185 – 1553.185	2.12170e-22
20.0	1538.185 – 1558.185	2.12171e-22
35.0	1530.685 – 1566.685	2.12171e-22
100.0	1498.185 - 1598.185	2.12172e-22
	$\sigma_I\lambda_o^2$ /c =	2.12172e-22

Notice how the results converge to the value given by (10) as the bandwidth increases. These values differ from the value obtained from equation (10) in the 3rd decimal place for the smallest bandpass and in the 4th place for bandpasses less than 4 Å in total width. Therefore, the simplifying assumptions that the radiation reaction term in the Abraham-Lorentz equation may be approximated

by a resistive term and that $\omega^2 - \omega_o^2 \approx 2\omega(\omega-\omega_o)$ are very good ones and testify to how sharply peaked the profile of the scattering coefficient is. That is, the function for the scattering coefficient is essentially a delta-function. So, when working with wavelength units in cm, one may use (10) for all practical bandpasses, but certainly not (9). Of course this depends on the precision with which one is working. Integration over bandpasses up to 500 Å wide were also carried out with no change in the result. The 35 Å-wide bandpass was the one used for the light curve analyzed by Pfeiffer *et al*. (2004). Actually this bandpass contains a doublet consisting of the above line separated from the other by about 2.6Å.

Table 2 presents similar results carried out for a visible bandpass, viz., for the Na I line at 5889.9504 Å with f =1.0. Clearly, the value for the integrated cross section depends on the resonant wavelength, when no simplifying assumptions are made in the solution of the Abraham-Lorentz equation or in the integration process and without introducing a broadening function.

Table 2

Values of the integrated scattering cross-section centered on the resonant wavelength 5889.9504 Å for Na I, for several different bandpasses and f = 1.0

Δλ(Å)	Bandpass (Å)	σ(cm^2-cm)
0.1	5889.9004 – 5890.0504	3.06916e-21
0.5	5889.7004 – 5890.2004	3.07044e-21
1.0	5889.4504 – 5890.4504	3.07067e-21
1.2	5889.3504 – 5890.5504	3.07071e-21
2.0	5888.9504 – 5890.9504	3.07078e-21
4.0	5887.9504 – 5891.9504	3.07084e-21
10.0	5884.9504 – 5894.9504	3.07087e-21
20.0	5879.9504 – 5899.9504	3.07089e-21
35.0	5872.4504 – 5907.4504	3.07089e-21
	$\sigma_I \lambda_o^2$ /c =	3.07090e-21

The result for a 25 Å wide bandpass for one of the components of the N V doublet at 1238.821Å is 1.35850e-22 cm^2-cm, which, again, is in numerical agreement with the value computed from equation (10). The latter bandpass will be used for the calculation of the N V density in the shock front of HD159176, which is presented below.

We have also numerically integrated (8) without any simplifying assumptions, such as γ is the same for all transitions. We have carried out the integration for various values of ω_o and various

bandwidths. Indeed, the results are independent of ω_o, to seven significant figures, when $d\omega$ is 10^7 Hz. or smaller.

Graphs of σ versus ω are presented by Jackson (1974, pg. 803), Rybicki and Lightman (1979, pg. 10) and Stone (1963, pg. 347) with relative scales. But these figures really do not convey how sharply peaked the function is, though Stone (1963, p. 347) attempts to quantify this. Table 3 gives computed values of σ for various wavelengths centered on the C IV resonant wavelength at 1548.185 Å, demonstrating this. Even a logarithmic scale on a graph would have difficulty showing this, since there are 8 orders of magnitude variation of σ over just 0.5 Å.

Table 3

Computed values for the scattering cross-section at several different wavelengths in the vicinity of the resonant wavelength of 1548.185 for Carbon IV

λ(Å)	σ(cm^2)
1547.685	1.59474e-18
1548.145	2.49103e-16
1548.155	4.42846e-16
1548.165	9.96392e-16
1548.175	3.98544e-15
λ_o=1548.185	1.14442e-10

We also attempted to see if *Mathematica* could find an analytical solution to (11). It could not. However, *Mathematica* did find an analytical solution when the *u* term in the scattering profile is that given by Mihalas and Padmanabhan, without making the simplifying assumption that ω^2-ω_o^2 may be approximated by $2\omega(\omega-\omega_o)$ The result is:

$$\sigma(\Delta\lambda) = 3\lambda_o^3\sqrt{M}\,\frac{\left\{\frac{Arctan\left[\frac{\sqrt{2M}\,x_f}{\sqrt{1-\sqrt{1-4M}-2M}}\right]}{\sqrt{1-\sqrt{1-\sqrt{1-4M}-2M}}}\right\}}{\pi\sqrt{2-8M}} - \frac{\left\{\frac{Arctan\left[\frac{\sqrt{2M}\,x_i}{\sqrt{1-\sqrt{1-4M}-2M}}\right]}{\sqrt{1-\sqrt{1-\sqrt{1-4M}-2M}}}\right\}}{\pi\sqrt{2-8M}}, \qquad (12)$$

where $M = 3\lambda_o^2/2\pi\sigma_T^2$ and $x = \lambda/\lambda_o$. The terms in the large curly brackets are the evaluation of the integral at the upper (x_f) and lower limits (x_i) of the integration. This solution yields numerical results in total agreement with the numerical integration of (11) to 6 significant figures? Again, this indicates that the exact express for *u* in the denominator of the scattering profile is not significant, except when working with very narrow lines.

Ion densities in Modeled Shocks

In addition to the determination of a numerical value for $n\sigma(\Delta\lambda)f$ through its use as an input parameter in the numerical integration of (5), it must also satisfy the relationship that involves the optical depth of the shock. That is, when computing the attenuation or eclipsing effects of the shock on the integrated fluxes, within the bandpass $\Delta\lambda$, emanating from the photospheres and winds of either star in the binary, one uses:

$$\tau_{\Delta\lambda} = nf\, t\int \sigma(\lambda)\emptyset(\lambda)d\lambda \tag{13}$$

Here $\emptyset(\lambda)$ is the usual broadening function. The number density of the ions is assumed to be constant over the thickness of the shock, t. Additionally, the correction for stimulated emission is assumed to be about 1, since shock densities are sufficiently low that most of the ions may be assumed to be in the ground state. Anyway, it is not possible to calculate what this factor should be. Since we are not interested in the details of a line profile, the broadening function may be replaced by simply dividing the result of the integration by the Doppler broadened width of the bandpass. This is necessary in order that the optical depth be dimensionless. Hence, (13) becomes:

$$\overline{\tau_{\Delta\lambda}} = nft\sigma_{\Delta\lambda}/\Delta\lambda \tag{14}$$

That is, one uses the mean value of the optical depth for the bandpass.

Hence, there are two constraints on the determination of the parameter $n\sigma_{\Delta\lambda}f$. The nominal value for t was $2R_{\odot}$ for the shock modeled for the EM Car system (Pfeiffer *et al*., 2004). Actually, the cross section in (5) and (14) is the weighted, combined cross section for scattering by the shock in the bandpass of a doublet, each component of which has essentially the same value for σ, but different oscillator strengths. The latter were taken from Cox (2000). The value of $\sigma_{\Delta\lambda}$ for each member of the doublet is numerically the value taken from Table 1 for the appropriate bandpass or computed from (10). The total width of the bandpass was determined to be the result of Doppler broadening by turbulence in the shock (Pfeiffer *et al*., 2004).

It is now realized that the value of $\sigma_{\Delta\lambda}f/\Delta\lambda$ used for calculating the ion density by Pfeiffer *et al*. (2004) was too small. For that paper, the value used for $\sigma_{\Delta\lambda}f/\Delta\lambda$ was 2.22 x 10^{-16} cm^2 for a 35 Å wide bandpass, whereas the value should have been 4.83 x 10^{-16} cm^2. Hence, the published value for the C IV number density of 1560 cm^{-3} is too large by a factor of about 2. The corrected value of the number density was then used to calculate the total mass density in the shock from

$$\rho = (1 + \frac{n_{He}}{n_H} A_{He} + \frac{n_{CIV}}{n_H} A_{CIV} + \frac{n_Z}{n_H} A_Z) n_H m_H \quad (13)$$

As a first approximation, we have assumed that all the carbon in the shock is entirely carbon IV. Values for the abundance ratios relative to hydrogen were taken from Cox (2000) to be 0.090 for helium and 3.63 x 10^{-4} for carbon. For the other metals, which are partially ionized to some unknown degree, we have taken a weighted mean value for A_z to be 9.00 and $\frac{n_Z}{n_H} = 0.0168$. The exact values are not significant, since n_z/n_H is so small. Now, since only n_{CIV} is known and not n_H, we may replace n_H outside the parentheses in (13) with (n_{CIV}/3.63 x 10^{-4}). This yields a mass density in the shock that is on the order of 10^{-18} g/cm^3. The shock in EM Car is located at a distance less than one stellar radius from both the primary and secondary stars. Typical wind densities in hot stars at photospheric distances of $r/R_* = 1$ are about 10^{-13} g/cm^3 (Lamers *et al*, p. 251). For close binaries, shocks are generally radiatively cooled. In such cases, there may be a factor of 10 to 100 increase in density from the pre-shock wind (Owocki, 2009). Furthermore, the densities found for the shocks in W-O-type binaries have been modeled by Gayley *et al*. (1996) to be on the order of 10^{-12} g/cm^3 or higher. Hence, one would expect the shock density in EM Car to be at least 10^{-12} g/cm^3. This means that the assumption that all the carbon is in the form of C IV is grossly wrong. Using the value 1 x 10^{-12} g/cm^3 for the density in (13) and solving for n_H, yields a value of 2.11 x 10^{12} cm^{-3}, which leads to n_C = 8.39 x 10^8 cm^{-3}! Therefore, most of the carbon in the shock is probably in the form of C V or higher states of ionization rather than C III. This is because there are no ostensible C III wind-lines noticeable in the *IUE* spectra, such as the doublet near 1428Å or the sextuplet at 1175 Å. This implies a very high temperature for the shock as is expected, since EM Car is a known X-ray source (Corcoran, 1996).

In the case of HD 159176, Pfeiffer *et al*. (1997) obtained an optical depth equal to 0.93 for the C IV in a modeled shock with nominal thickness $2R_\odot$. The result for the C IV density in the shock as reported in that paper (n=0.13 cm^{-3}) is also incorrect because of an incorrect value used for $\sigma_{\Delta\lambda} f/\Delta\lambda$, namely 8.0 x 10^{-11} cm^2. The correct value for this parameter, as given above, leads to a C IV number density equal to 1.48 x 10^4. For the N V doublet, the above authors obtained an optical depth of 1.01, which leads to an ion number density of 2.85 x 10^4 cm^{-3}. Now the stellar components in HD 159176 are hotter than those in EM Car. So, in light of the previous discussion for the shock density in EM Car, it may be concluded that the total mass density in the shock of HD 159176 is greater than the shock in EM Car. A comparison of the C IV densities of the two systems would not yield an estimate of the density ratio of the shocks in these systems, since the ionization temperatures are not well determined.

A preliminary investigation of the IUE spectra for the contact binary TU Miscue has found no evidence of a shock in this system. This is probably the result of the contact configuration of the binary. That is, radiation breaking (Gayle *et al*., 1996) and the close proximity of the stars does not permit the winds to accelerate to sufficient speeds in the vicinity of the stars, where the wind density is sufficiently high, that an interaction of the winds would produce a detectable enhancement in the density.

Conclusions

The atomic resonant scattering profile is so sharply peaked, that the approximations made in most texts for the solution of the Abraham-Lorentz equation are very good ones to 5 significant figures, when integrating the cross section over bandpasses greater than 2Å in width. For bandpasses narrower than 2 Å, down to 0.50Å, the assumptions leading to (10) are even good to 4 significant figures. One certainly does not want to use the value given by (9) for the cross section for all bandpasses, since the integrated value of σ_I is definitely dependent on the resonant wavelength, independently of a broadening function. Oddly though, we found that σ_I is independent of the resonant frequency ω_o for transitions ranging from the ultraviolet to the near infrared. However, these matters are not presented uniformly in most textbooks.

The C IV densities found for the shock fronts in EM Car and HD159176 independently indicate very high ionization temperatures in both systems and a much higher density in the shock of HD 1591756 than in EM Car.

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