

CHAPTER 6

Stellar Evolution

6-1 Introduction

Stars evolve in the sense that they pass through different stages of a stellar life cycle that is measured in billions of years. The longer the amount of time a star spends in a particular stage of evolution, the greater the number of stars that one observes in that stage.

Stars evolve at different rates and they pass through different sequences of evolutionary stages, depending on their mass. **In general, the greater the mass of a star, the faster it evolves.** For example, red dwarf stars may spend more than a hundred billion years as a main sequence star, whereas the much more massive stars, which are at the top of the main sequence, spend only a few million years as a main sequence star. Stars with a mass similar to that of the Sun have a main sequence lifetime of about 10 billion years.

Objects with a mass less than about 0.06 solar mass units (the **Kumar Limit**) never initiate TNF of any significant amount, and never become true stars. These objects are called **Brown Dwarfs**. Since brown dwarfs are intrinsically faint objects it is difficult to detect them at great distances. However, several nearby ones have been detected recently, so their existence has been verified.

Brown dwarfs, like ordinary stars, are comprised mostly of hydrogen and helium, whereas terrestrial-like planets are comprised mostly of metals and silicates (rocks). Jupiter Saturn Uranus and Neptune are comprised mostly of hydrogen and helium just like stars and brown dwarfs. The IAU has established a lower limit of 17 Jupiter masses to qualify as a brown dwarf. Such an object could not even initiate deuterium fusion, whereas a brown dwarf could.

The exact sequence of evolutionary stages also depends on the mass of a star.

6-2. The Russell-Vogt Theorem

The structure of a star is uniquely determined by its mass and molecular weight *vis-à-vis* the law of gravity and the laws of thermodynamics. This is the Russell-Vogt Theorem. Therefore, stars with the same molecular weight but different mass will define a locus in a temperature-luminosity diagram, that is, the ZAMS. As the average molecular weight changes due to TNF, the evolution of the star proceeds, the rate being determined by the mass via gravity.

Gravity is a force of attraction on every little parcel of the star acting as if all the mass of the star were a point at the very center of the star. This force acts to contract and compress the star. The greater the mass of a star, the stronger gravity is and the greater the compression of the star. The greater the compression of the star, the higher the temperature becomes. The higher the temperature is, the greater the gas pressure. The gas pressure produces a force that is trying to expand the star and balance gravity. Also, the higher the temperature, the faster the TNF reactions take place. The faster the TNF reactions take place, the faster the star evolves.

Hence, stellar evolution is the result of the interplay between gravity and gas pressure.

6-3. Stages of Stellar Evolution

Gravitational contraction is what begins the formation of a star and its evolution. As a star contracts, the virial theorem (see appendix) says, that for a gravitationally bound system, half the change in potential energy goes into kinetic energy of motion of the particles (atoms, molecules, and dust). As the particles collide, this kinetic energy is randomized and becomes thermal energy, E_T , that may be associated with a kinetic temperature. Conservation of energy demands that the other half of the potential energy becomes electromagnetic energy or radiation, E_R .

$$\Delta GPE \rightarrow \frac{1}{2}E_T + \frac{1}{2}E_R$$

Since energy is being lost through radiation, the gas pressure can never keep up with gravity to bring about hydrostatic equilibrium. At best, the star is in a state of quasi-hydrostatic equilibrium. So the star can not stop contracting if it depends totally on its gravitational potential energy to produce its gas pressure. This assumes that the gas is infinitely compressible, such as an ideal gas, and that there is no change in phase. However, ionization and nuclear forces eventually come into play to change matters.

A. Nebular Stage

Stars form from gas and dust clouds called nebulae, when gravity dominates over gas pressure. The cloud begins to contract and heat up. Random motions within the cloud become organized and the cloud begins to rotate.

We now investigate further the energetics of a collapsing cloud and what the criteria are for such a collapse to occur. We start by considering the gravitational potential energy of a gravitationally bound system such as a star. Imagine such a star is assembled by bringing a mass element dm_i from a distance $r = \infty$ to the surface of an already assembled mass M_r that lies within a radius r .

Then $U_g = \int_r F_g dr$, where F_g is the force of gravity due to M_r on dm_i , that is

$$F_i = GM_r dm_i / r^2$$

and

$$dU_i = -GM_r dm_i / r$$

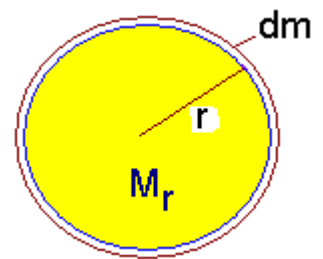
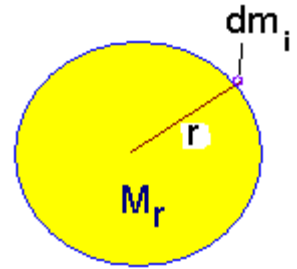
is the differential change in potential of the assembly when dm_i is added to M_r . The change in the potential is negative, that is, the potential energy decreases. Assume many dm_i are added to M_r to make up a spherical shell of thickness dr and volume $4\pi r^2 dr$ with uniform mass density ρ . The mass of this shell, dm , is the sum of all the dm_i values or

$$dm = \sum dm_i = 4\pi r^2 \rho dr$$

The differential change in the gravitational potential of the growing star by adding the layer dr is:

$$dU_g = -GM_r(4\pi r^2 \rho) dr / r$$

We now integrate over all mass shells from $r = 0$ to $r = R$, where R is final radius of the star.



$$U = \int dU = -4\pi G \int_0^R M_r \rho r dr. \quad (6-1)$$

If we make the simplifying assumption that the density is constant and equal to the mean value, $\bar{\rho} = M_*/[(4/3)\pi R_*^3]$, then $M_r = (4/3)\pi \bar{\rho} r^3$. So (6-1) becomes:

$$U = -4\pi G \int_0^{R_*} \left(\frac{4}{3}\pi \bar{\rho} r^3\right) \{M_*/[(4/3)\pi R_*^3]\} r dr,$$

which simplifies to

$$U = \frac{-4\pi G M_* \bar{\rho}}{R_*^3} \int_0^{R_*} r^4 dr.$$

Substituting for $\bar{\rho}$ and simplifying and integrating we obtain:

$$U = \frac{-3GM_*^2}{R_*^6} \frac{r^5}{5} \Big|_0^{R_*} = \frac{-3GM_*^2}{R_*^6} \frac{R_*^5}{5} = \frac{-3GM_*^2}{5R_*} \quad (6-2)$$

Now the virial theorem (See notes in seminar room) states that the kinetic energy of a system is $-(1/2)$ the potential energy. Alternatively, the total energy of a system, that is, the sum of the kinetic and potential energy, is one half the potential energy. The virial theorem applies to a wide range of systems, from an ideal gas to a cluster of galaxies. Hence, for our collapsing cloud:

$$E_{Tot} = \left(\frac{1}{2}\right) U = \frac{-3GM_*^2}{10R_*} \quad (6-3)$$

Note that the energy is negative. This is because the system is gravitationally bound. If the radius of the star is larger, the total energy is less negative and therefore greater. Therefore, as a cloud or star contracts, it loses energy. It does so by radiating as an incandescent body. For a star in hydrostatic equilibrium, the total energy is not changing.

We now calculate the energy lost by the Sun as it is collapsing from an extremely large cloud of gas and dust to its present size. The energy lost is the difference between its initial energy, when R is essentially infinite, and its final energy, when R_\odot is the present radius of the Sun.

$$\Delta E = E_i - E_f = - (E_f - E_i)$$

$$\Delta E = \frac{-3GM_*^2}{10} \left(\frac{1}{R_\odot} - \frac{1}{R_i} \right)$$

But $1/R_i$ is essentially zero, so $\Delta E = -E_f$, or:

$$\Delta E = \frac{-3GM_*^2}{10} \left(\frac{1}{R_\odot} \right) = 1.10 \times 10^{48} \text{ ergs.}$$

If we assume the Sun's luminosity has remained constant over the amount of time the Sun has been contracting, then this would set an upper limit on how long the Sun could last this way. Namely

$$t = \Delta E / L_{\odot} = 10^7 \text{ years.}$$

This is totally inadequately to explain the fact that paleontological records indicate that the Sun's luminosity has been essentially constant for 4.5×10^9 year.

From kinetic theory, the mean kinetic energy per particle in a gas is $(3/2)kT$. If there are N particles in a protostar cloud, then the total kinetic energy, K , of the cloud is $(3/2)NkT$, where $N = M_c/\mu m_H$ and M_c is the total mass of the cloud. By the virial theorem, the condition necessary for a system to be in hydrostatic equilibrium is $2K+U=0$. For a cloud to undergo gravitational collapse, it must be that $2K < |U|$, or

$$\frac{3M_c kT}{\mu m_H} < \frac{3}{5} \frac{GM_c^2}{R_c} \quad (6-4)$$

Now simplify (6-4) and solve for M_c :

$$\frac{5R_c kT}{G\mu m_H} < M_c \quad (6-5)$$

Let ρ_o be the initial density of the cloud, where

$$\rho_o = \frac{M_c}{\frac{4}{3}\pi R_c^3} \quad (6-6)$$

Then

$$R_c = \left(\frac{3M_c}{4\pi\rho_o} \right)^{1/3} \quad (6-7)$$

Substitute (6-7) into (6-5):

$$\begin{aligned} M_c &> \frac{5kT}{G\mu m_H} \left(\frac{3M_c}{4\pi\rho_o} \right)^{1/3} \\ \frac{M_c}{M_c^{1/3}} &> \frac{5kT}{G\mu m_H} \left(\frac{3}{4\pi\rho_o} \right)^{1/3} \\ M_c^{2/3} &> \frac{5kT}{G\mu m_H} \left(\frac{3}{4\pi\rho_o} \right)^{1/3} \\ M_c &> \left(\frac{5kT}{G\mu m_H} \right)^{3/2} \left(\frac{3}{4\pi\rho_o} \right)^{1/2} \end{aligned} \quad (6-8)$$

This was first derived by Sir James Jeans, a British astrophysicist and is known as the Jeans criterion. A mass equal to the quantity on the right of (6-8) is called the Jeans mass, M_J . Hence, for a protostar cloud of initial mass M_c to undergo spontaneous collapse, $M_c > M_J$.

Now substitute the expression for the Jeans mass into (6-7):

$$R_c = \left(\frac{3}{4\pi\rho_o} \right)^{1/3} \left[\left(\frac{5kT}{G\mu m_H} \right)^{3/2} \left(\frac{3}{4\pi\rho_o} \right)^{1/2} \right]^{1/3}$$

$$R_c = \left(\frac{3}{4\pi\rho_o}\right)^{1/2} \left(\frac{3}{4\pi\rho_o}\right)^{1/6} \left(\frac{5kT}{G\mu m_H}\right)^{3/6}$$

$$R_c = \left(\frac{3}{4\pi\rho_o}\right)^{1/2} \left(\frac{5kT}{G\mu m_H}\right)^{1/2}$$

$$R_c = \left(\frac{15kT}{4\pi\rho_o G\mu m_H}\right)^{1/2} = R_J \quad (6-9)$$

This is the critical radius for a cloud to collapse and is known as the *Jean's Length*.

For example, suppose we have a typical diffuse cloud with $T = 50\text{K}$, and $n = 500 \text{ cm}^{-3}$. Assume the cloud is composed of H I and $\mu = 1$. Then $\rho_o = m_H n_H = 8.4 \times 10^{-22} \text{ g/cm}^3$. Substituting these values into (6-8) gives $M_J \cong 1500 M_\odot$. This exceeds the 1 to 100 M_\odot mass range that has been estimated for the H I clouds that are observed in the galaxy. Hence, they are stable against collapse. On other hand, the dense cores of giant molecular clouds have $T = 150 \text{ K}$ and $n = 10^8 \text{ cm}^{-3}$, implying $\rho_o = 2 \times 10^{-16} \text{ g/cm}^3$ and therefore $M_J = 17 M_\odot$. This is well within the observationally determined mass range of 1- $10^3 M_\odot$ for these objects. Hence, these clouds are undergoing gravitational collapse.

As a massive cloud contracts, the density increases but the temperature remains constant, since energy is radiated away. Hence the value of M_J for the cloud decreases according to equation (6-9). Any inhomogeneities in the cloud could eventually exceed M_J and these knots in the cloud would collapse separately and locally. This would cause fragmentation of the cloud which can then cascade producing many smaller collapsing clouds with masses in the stellar range. Stars would then form in clusters as we observe. If the collapse becomes adiabatic because of opacity, then T rises and fragmentation stops.

B. Dark globule Stage

The stellar nebula gravitationally contracts, gets denser and becomes opaque. They are then observed silhouetted against a background of fluorescing gas. These objects are also called Bok globules in honor of the astronomer who first interpreted what they are correctly. The globules can be seen in the adjacent photo.



Photo courtesy of NASA.

C. Protostar or Pre-main sequence Stage

Protostars are stars evolving towards the main sequence. This evolution may be divided into two main phases, which we shall refer to as A and B.

A. Dynamic Collapse. This is a stage of rapid contraction. Initially the star is an opaque cloud of material in free fall. Not all gravitational potential energy can go into thermal energy to raise temperature. Instead, the energy is used to dissociate molecules such as H_2 , evaporate the dust particles, and ionize the atoms. γ is $4/3$

The star is unstable.

Stellar densities finally achieved in $\sim 10^3$ years. This stage lasts as long as the radius of the star is greater than R_c , where R_c is actually the radius of a core.

$$R_c = \frac{43.2 (M_*/M_\odot)}{1 - 0.2X} \text{ (solar radii)}$$

Whithin R_c , the gas is completely ionized. Once R_c is attained, the internal temperature rises and $\gamma = 5/3$. This happens in time Δt

$$\Delta t = 1.6 \times 10^7 \left(\frac{M_*}{M_\odot} \right)^2 \left(\frac{R_\odot}{R_*} \right) \left(\frac{L_\odot}{L_*} \right) \text{ years}$$

The core temperature is

$$T_c \approx 3.7 \times 10^5 \mu (1 - 0.2X) \text{ Kelvins}$$

Since L is large, Δt is relatively short.

For the Sun, $R_c \approx 50 R_\odot$

For the Sun, the core grows by accretion to $0.5M_{\odot}$ in about 10^5 yrs. after the initial core is formed. It takes $\sim 10^6$ years for accretion to increase the mass of the core to $\sim 1M_{\odot}$ to become the full grown Sun.

During the dynamic collapse the star becomes a Herbig-Haro object. These objects are self-luminous, have a reasonably round appearance but fuzzy, and change their appearance over several years. The interior is about 10^6 K. Radiate mostly IR from a 10^3 K surface.

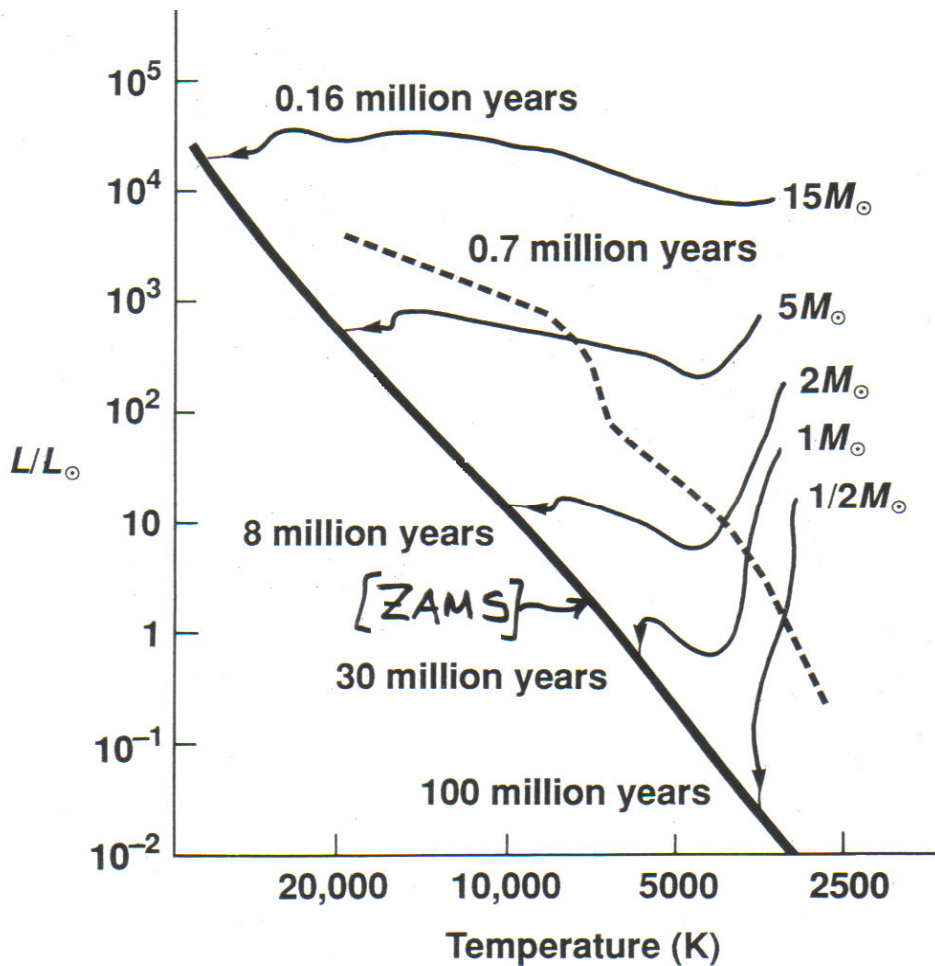
B. Slow contraction phase: A series of quasi-hydrostatic equilibrium states or Kelvin-Helmholtz contraction where

$$GPE \longrightarrow \frac{1}{2} (\text{thermal energy}) + \frac{1}{2} (\text{radiation})$$

Protostars remain opaque until they get hot enough that their radiation ionizes the gases and vaporizes the dust particles in the cocoon from which the object formed. It then begins to shine as a star. Stars identified in this later stage are called T Tauri stars. Such stars are said to be in a state of quasi-hydrostatic equilibrium: they can not stop contracting because they lose heat by radiation faster than they can generate heat by contraction.

There may be some deuterium fusion taking place in the core of the star at this time.

Hayashi Tracks



The temperature-luminosity diagram to the left shows pre-main sequence evolutionary tracks computed by Hayashi for several different masses. These tracks are for phase B of the protostar stage. The dashed curve indicates the onset of the T Tauri stage.

T Tauri Stage

This is the first true stellar stage but such stars are still living on GPE. The star now has a luminosity output that is sufficient to vaporize the dust and molecular cloud in which it is embedded. The star now begins to shine like a star,

though it may do so erratically as various clouds of high opacity continue to revolve around the star. The spectra of such stars show strong emission lines indicative of a circumstellar environment. In addition there are absorption lines due to lithium, indicative of their young age. The first star identified to be in this stage of evolution was the variable star T Tauri which serves as the prototype star for this class. Such stars are slowly contracting towards the main sequence in the H-R Diagram.

There is observational evidence for the existence of disks of gas and dust surrounding young stars. These disks are the remnants of the clouds from which the star has formed by collapse. More than likely, planets are growing by accretion of dust particles in such disks. An example is shown for the star Beta Pictoris below. The disks of the bright stars have been occulted in order to reveal the faint image of the disk.

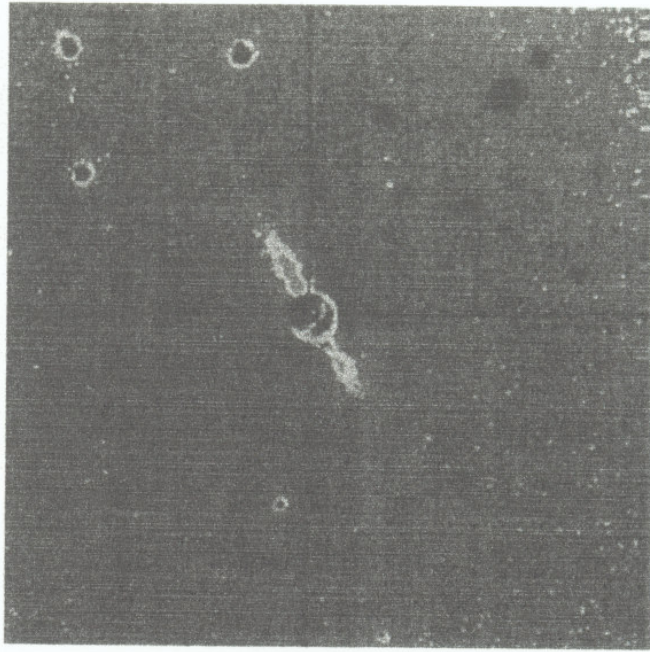


Figure 6-5. An infrared image of β Pictoris, showing its circumstellar disk. (Courtesy of NASA/JPL.)

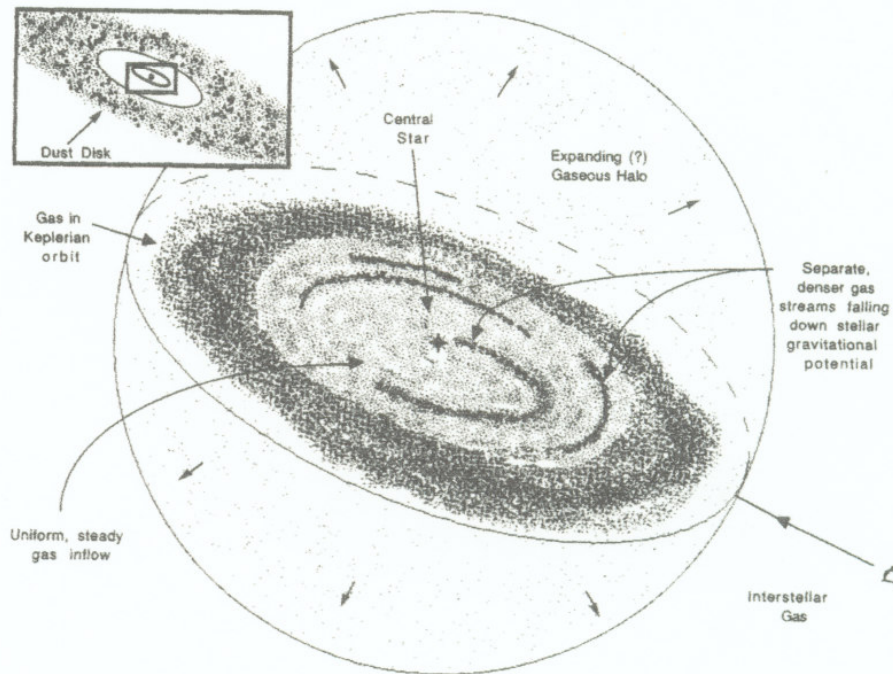


Figure 6-6. An artist's conception of the β Pictoris system. Clumps of material appear to be falling into the star at the rate of 2 or 3 clumps per week. Some matter may also be leaving the system as an expanding halo. (Figure adapted from Boggess et al., *Ap. J. Lett.*, 377, L49, 1991.)

Expansion of the disk is caused by radiational heating due to the radiation from the newly formed star.

D. Main Sequence Stage

A star enters this stage when it initiates TNF of hydrogen nuclei into helium nuclei in its core. This happens when the core of the star reaches about 5 million Kelvins. Main sequence stars are characterized by:

- (1). TNF of H into He in the core of the star.
- (2). Hydrostatic equilibrium: At every point in the star, gravity is balanced by gas pressure. This happens because the star does not radiate away its heat at the surface faster than it generates it by TNF.

Stars that are beginning the main sequence stage define in the H-R diagram what is called the Zero Age Main Sequence or ZAMS.

Table 1

Main sequence lifetimes:

$15.0M_{\odot}$: 10 million years

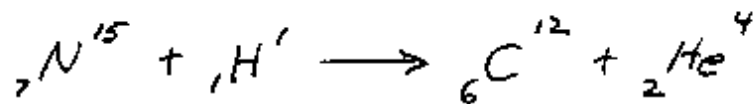
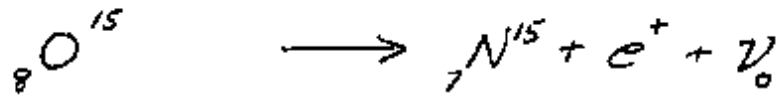
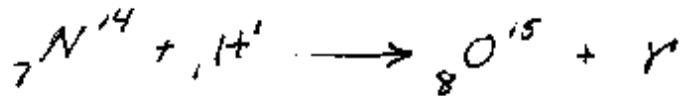
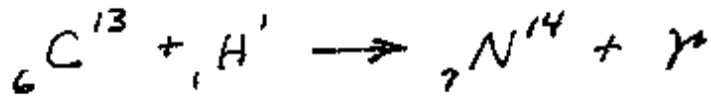
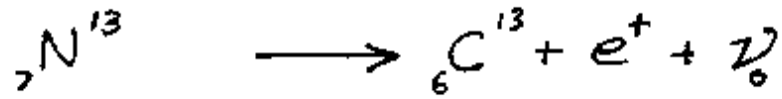
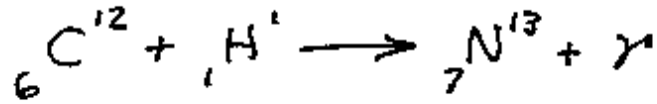
$5.00M_{\odot}$: 65 million years

$2.25M_{\odot}$: 480 million years

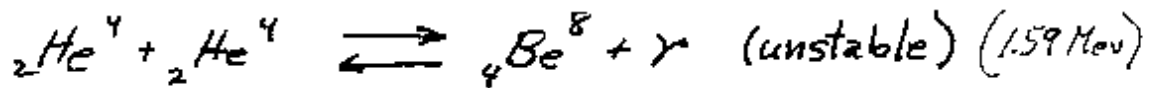
$1.00 M_{\odot}$: 7 - 9 billion years

$0.10 M_{\odot}$: 100 billion year

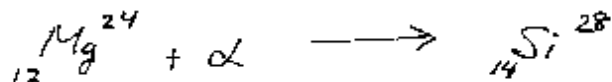
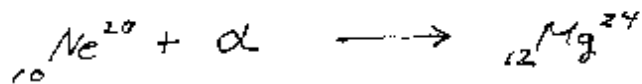
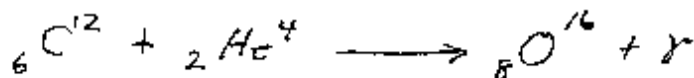
Carbon Cycle of H Fusion Dominates for $T > 20 \times 10^6 \text{K}$



Triple α Reaction at $100 \times 10^6 \text{K}$



Before a Be^8 can decay it may be hit by an α -particle (a helium nucleus). Then



E. Shell-hydrogen "Burning" stage

Upon exhaustion of H in the core, the star begins to contract and get hotter. The star has now left the main sequence. Eventually TNF of H into He begins in a shell around the core. The energy released from this shell heats the outer layers of the star thereby causing these layers to greatly expand. The star then tracks to the right in the H-R Diagram, getting cooler and larger. Meanwhile, the He core continues to contract and get hotter.

F. Red Giant or Supergiant

The star now begins to expand enormously as the temperature in the core increases and provides more radiation to be absorbed by the envelope. Which it becomes depends on the mass of the star.

G. He Flash.

At 100 million Kelvins, He nuclei in the core collide and fuse to form C by what is known as the triple-alpha reaction. The onset of He fusion happens almost explosively, and the star undergoes a rapid change in its structure to enter a stage of hydrostatic equilibrium again as it undergoes helium fusion. The star then descends in the H-R Diagram to become a horizontal branch star

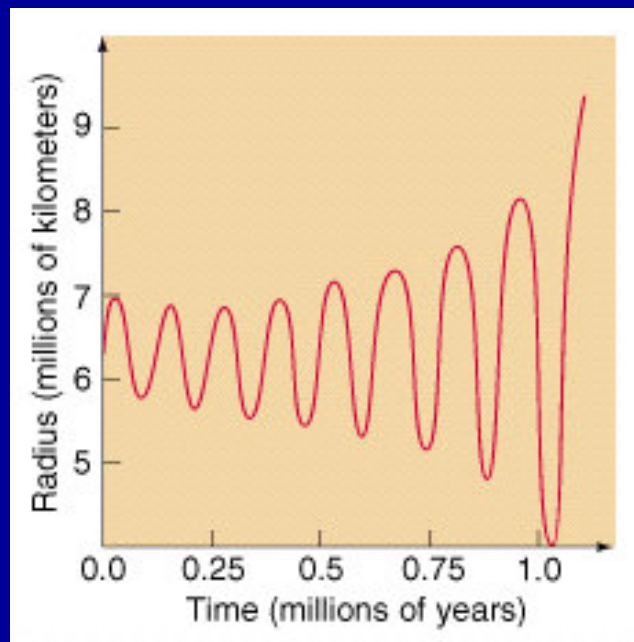
H. Horizontal Branch Stage

Which is also the stage of He fusion in the core with hydrostatic equilibrium again, similar to the main sequence stage. This is sometimes referred to as the helium main sequence stage. In addition to the production of C, the elements O, Ne, Mg, and Si are produced. See the previous page for a list of these reactions.

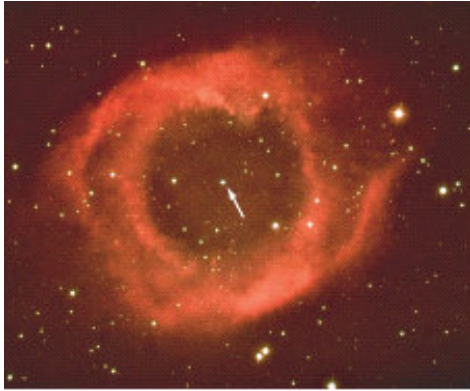
I. Asymptotic Giant Branch Stage

When all the He is exhausted in the core, the core begins to contract again until He fusion occurs in a shell around the core, while hydrogen fusion occurs in a layer above that. The star then expands to become even larger, while the core is contracting. The star is then said to ascend the asymptotic giant branch

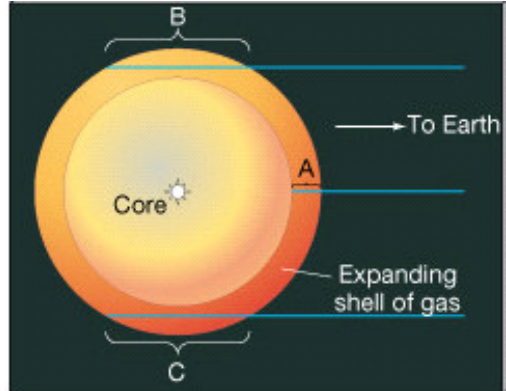
In stars like the Sun, electron pressure will eventually halt further contraction of the star and it eventually enter the "Planetary Nebula Stage." This is a stage of instability due to He fusion in a shell around the carbon core. This instability causes the outer envelope to undergo pulsations of ever increasing amplitude until the outer layer is lost to the star's gravity. The outwardly expanding envelope then forms a "planetary nebula." This term is an accident of history and has nothing to do with planets, but merely the telescopic appearance of the expand shell around the star.



The Planetary Nebula Stage



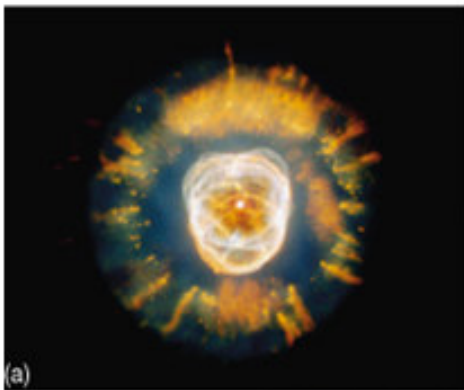
(a)



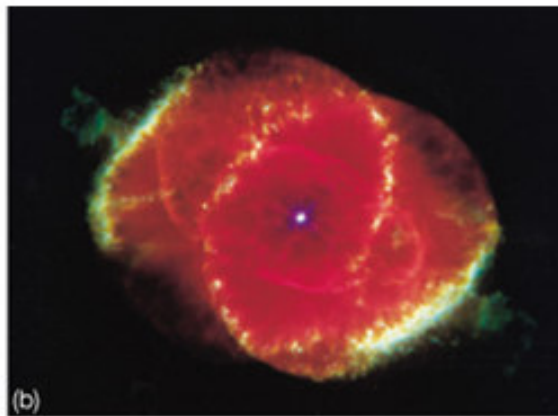
(b)



(c)



(a)



(b)



J. Core carbon “burning” Stage

When the core temperature reaches 600 million Kelvins, carbon nuclei begin to undergo fusion into Mg. The star then readjusts itself and hydrostatic equilibrium is established again but this does not last very long.

K. Other stages of TNF

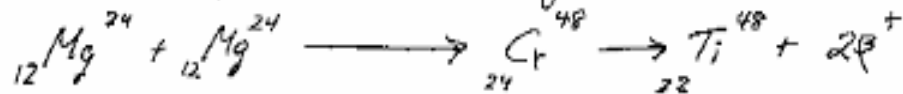
The above scenarios repeat as the elements in the core are exhausted and the core contracts to a higher temperature and a new TNF reaction begins. This is the end though, since Fe will not undergo TNF. What happens to a star after developing an iron core will be taken up later.

Other Reactions

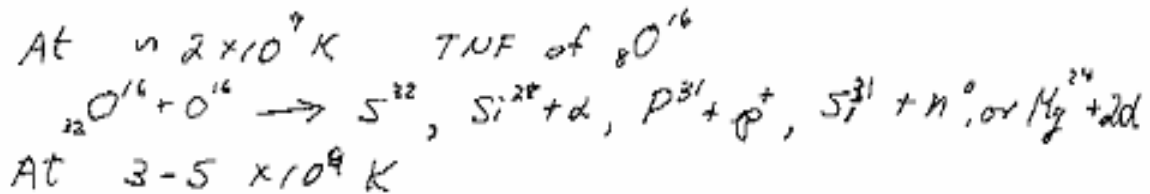
At $600 \times 10^6 \text{ K}$, TNF of C^{12} occurs



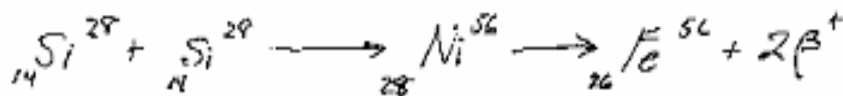
At $1 \times 10^9 \text{ K}$, TNF of ${}^{12}_{12}\text{Mg}^{24}$ occurs



At $\sim 2 \times 10^9 \text{ K}$ TNF of ${}^8_8\text{O}^{16}$



At $3-5 \times 10^9 \text{ K}$



Post Main Sequence Evolutionary Tracks

We now turn to looking at the post main sequence evolutionary tracks that have been computed by I. Iben. These are shown in the next figure for several different masses. Following the figure is a table that lists the various amounts of time a star takes to go from one numbered point on the track to the next for the different masses. The figure and table are taken from *The Annual Reviews of Astronomy and Astrophysics*, **5**, 570ff.

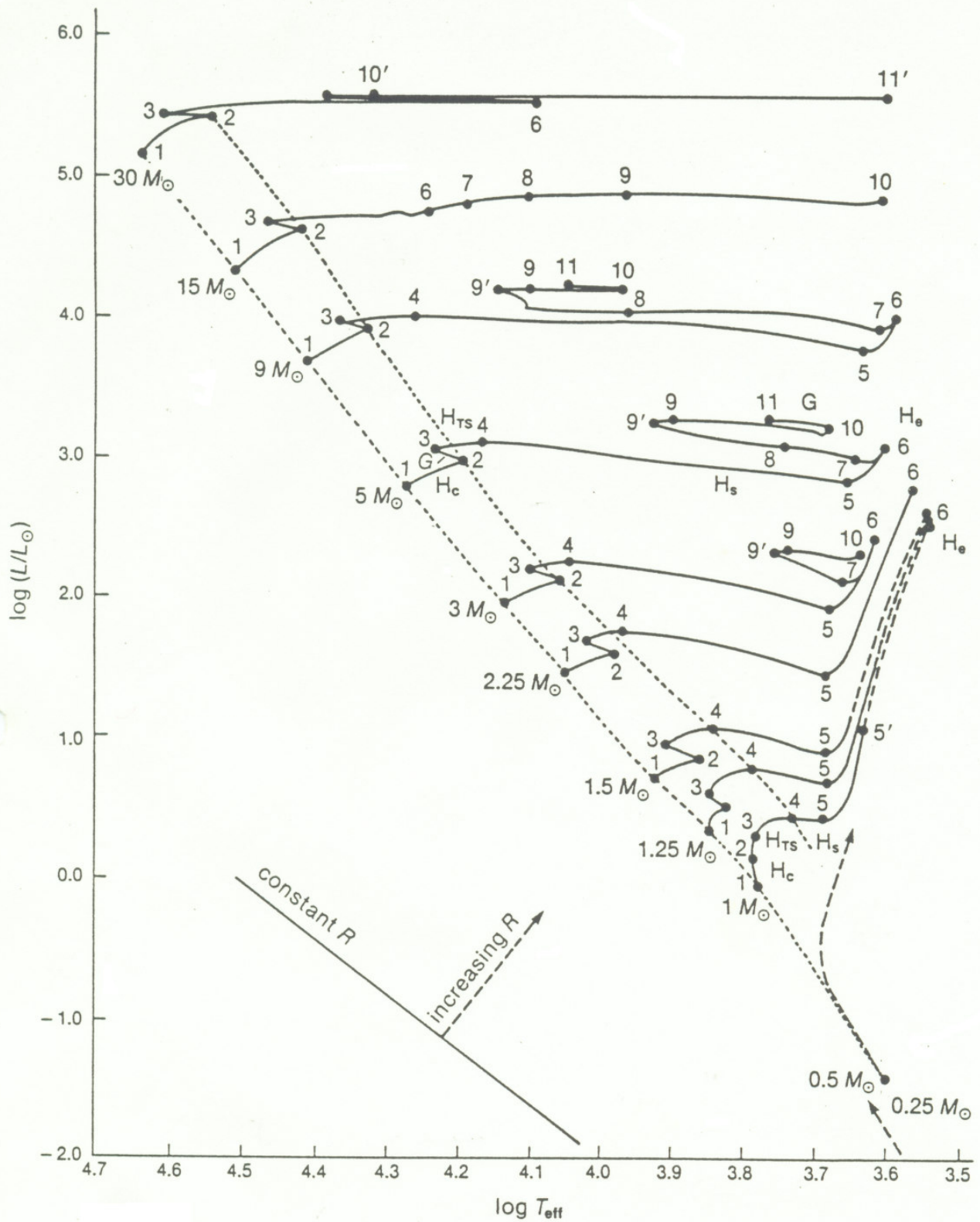


Figure 6-7. Evolutionary tracks for models of stars after the main sequence. Model mass is shown next to the initial point on zero age main sequence. Dotted lines indicate boundaries of the main sequence. Lines of constant radius and increasing radius as shown in lower left. Elapse times between points are shown in Table 3. The stages are labeled as: H_c , hydrogen core burning; H_{TS} , thick hydrogen shell burning; H_s , shell hydrogen burning; He , helium core burning; and G , gravitational energy release. The $15 M_\odot$ track does not reverse in the giant region, because the semiconvective region was treated as fully convective in this model.

The table below gives the elapsed time in years between the numbered points on the evolutionary tracks depicted in the figure above. The numbers in the parentheses to the left of the table entries gives the power of ten that the entry must be multiplied.

Table 3

Mass (M_{\odot})	Interval				
	1-2	2-3	3-4	4-5	5-6
30	4.80 (6)	8.64 (4)		~1.0 (4)	
15	1.010 (7)	2.270 (5)		7.55 (4)	
9	2.144 (7)	6.053 (5)	9.113 (4)	1.477 (5)	6.552 (4)
5	6.547 (7)	2.173 (6)	1.372 (6)	7.532 (5)	4.857 (5)
3	2.212 (8)	1.042 (7)	1.033 (7)	4.505 (6)	4.238 (6)
2.25	4.802 (8)	1.647 (7)	3.696 (7)	1.310 (7)	3.829 (7)
1.5	1.553 (9)	8.10 (7)	3.490 (8)	1.049 (8)	~2 (8)
1.25	2.803 (9)	1.824 (8)	1.045 (9)	1.463 (8)	~4 (8)
1.0	7 (9)	2 (9)	1.20 (9)	1.57 (8)	~1 (9)

The following refers to Figure 6-7

Point	Stage of Evolution
1	Zero Age Main Sequence
2 - 4	Evolution on the main sequence
4	End of main sequence stage
5	Shell hydrogen burning
5 - 6	Ascent of the giant branch
6	Helium flash
7 - 9'	Core He burning stage and hydrostatic equilibrium.
9	End of core He burning stage
9 - 10	Asymptotic giant branch stage, shell-helium burning.

The following diagram shows the evolution a star similar to the Sun. The scale's are approximate. Iben's track for a 1.0 solar mass star terminates at the helium flash.

