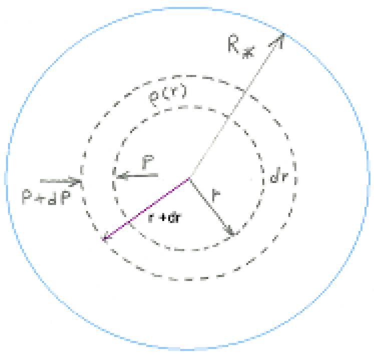
CHAPTER 5
Stellar Interiors

5-1. Hydrostatic Equilibrium

Hydrostatic equilibrium is a state or condition where, at every point in a star, gas pressure pushing outwards is balanced by gravity pulling inwards. Hence, the star is neither expanding nor contracting. We shall now develop the equations that express this balance. See the schematic below and consider an arbitrarily located, spherically symmetric layer or shell of thickness dr



$$A = 4\pi r^2$$

$$V = 4\pi r^2 dr$$

$$dM = 4\pi r^2 \rho(r)\, dr$$

Gravity at point r

$$F = \frac{G M(r)\, dM}{r^2} \qquad (5\text{-}1)$$

Net pressure:

$$P - (P + dP) = -dP \qquad (2)$$

This is outwards.

The force outwards is:

$$F = P \times A = -4\pi r^2 dP \qquad (3)$$

This is balanced by gravity inwards:

$$-4\pi r^2 dP = G M(r)\, dM / r^2 = G M(r) [4\pi r^2 \rho(r)\, dr]$$

$$\frac{dP}{dr} = \frac{-G M(r)\, \rho(r)}{r^2} \qquad (4)$$

Also:

$$dM(r) = 4\pi r^2 \rho(r)\, dr \qquad (5)$$

<<R_*. Then

5-2. The Equation of State

For most stars, we can assume the ideal gas law is valid, viz.,

$$P(r) = n(r)kT(r) \qquad (5\text{-}6)$$

Here n(r) is the total number density of all atoms. If $\mu(r)$ is the molecular weight, then

$$n(r) = \rho(r)/\mu(r)m_H \qquad (5\text{-}7)$$

Then

$$P(r) = \rho(r)kT(r)/\mu(r)m_H \qquad (5\text{-}8)$$

5-3. Molecular Weight

We now address the meaning of molecular weight in stars:

Stars are comprised of a mixture of a various ions and electrons, each with a mass m_i. Each particle has an abundance by weight a_i, which is dimensionless and less than 1. The mass density of a species is $\rho_{i,}$ and the number density n_i. Then $\rho_i = m_i n_i = a_i \rho$, where ρ is the total mass density of the gas.

We now define the mean molecular weight of the gas as

$$\mu = \langle m \rangle / m_H \qquad (5\text{-}9)$$

Here $\langle m \rangle$ is mean or average mass of a gas particle. In general, $\mu = \mu(r)$, but we shall drop this radial dependence in our further notation. The mean molecular weight is the average mass of a gas particle in units of the mass of the hydrogen atom or proton. The value of μ depends on the chemical composition of the gas and the state of ionization of each atom comprising the gas. Ionization is important because we must include electrons in computing the average mass per particle, $\langle m \rangle$.

Let us first consider a completely neutral gas, then

$$\langle m_o \rangle = (\Sigma\, m_i n_i / \Sigma n_i), \qquad (5\text{-}10)$$

where the subscript "o" refers to a neutral gas. If we divide the numerator by $m_{H,}$ we get μ. Define the atomic weight, A_i , to be m_i/m_H, then (5-9) becomes:

$$\mu_o = (\Sigma\, A_i n_i / \Sigma n_i). \qquad (5\text{-}11)$$

For a completely ionized gas, with atoms of atomic numbers Z_i and weights $A_i = m_i/m_H$, the mean molecular weight is

$$\mu_{ion} = \Sigma A_i n_i / \Sigma_{ni}(1+z_i), \qquad (5\text{-}12)$$

where z_i is the number of free electrons that result from ionizing atom i.

Now we introduce , mass fractions X_Z, where Z is atomic number:

For Z =1, hydrogen, $X_1 = X$ = Total mass of hydrogen/Total mass of gas .

For Z=2, helium, $X_2 = Y$ = Total mass of helium/Total mass of gas.

For Z > 2, "metals", Xz = Z = Total mass of metals/Total mass of gas.

Let N_i be the total number of atoms of species i that are in a sample of the gas (it could be number density). Then

$$X_i = N_i m_i / \Sigma\, N_i m_i \qquad (5\text{-}13)$$

An alternate expression for mean molecular weight is often used. Starting with (5-9) we have $\langle m \rangle = \mu m_H$. Then

$1/\mu m_H = \Sigma N_i / \Sigma N_i m_i$ = (Total number of atoms of type i)/ (Total mass of gas). (5-14)

Now multiply the terms in the summation in the numerator by $(N_i m_i)/(N_i m_i)$ and rearrange.

$$1/\mu m_H = \frac{\sum_i N_i (N_i m_i)/(N_i m_i)}{\sum_i N_i m_i} = \sum_i \frac{N_i}{N_i m_i}\left[\frac{N_i m_i}{\sum_i N_i m_i}\right] = \sum_i \frac{N_i}{N_i m_i} X_i \qquad (5\text{-}15)$$

Now substitute that $m_i = A_i m_H$ into (5-15):

$$\frac{1}{\mu m_H} = \sum_i \frac{N_i}{N_i A_i m_H} X_i = \sum_i \frac{X_i}{A_i m_H} \qquad (5\text{-}16)$$

Finally, we solve for $1/\mu$ and get:

$$\frac{1}{\mu} = m_H \sum_i \frac{X_i}{A_i m_H} = \sum_i \frac{X_i}{A_i} \qquad (5\text{-}17)$$

Now consider a neutral gas. Recall that $A_i = m_i/m_H$ so the atomic weight for H is 1 and for He it is 4. Then

$$\frac{1}{\mu_o} = X + \frac{1}{4} Y + \left\langle \frac{1}{A} \right\rangle_o Z \qquad (5\text{-}18)$$

It has been determined that a good value of $\langle 1/A \rangle_o$ is 1/15.5. For a completely ionized gas we have

$$\frac{1}{\mu_{ion}} = \sum_i \frac{1+z_i}{A_i} X_i \cong 2X + \frac{3}{4} Y + \left\langle \frac{1+z}{A} \right\rangle_{ion} Z\,, \qquad (5\text{-}19)$$

for i =3 and only hydrogen, helium, and all metals are lumped together as Z. For the term $\left\langle \frac{1+z}{A} \right\rangle_{ion}$, the average value is about 1/2. Verify this. So, in general, we may write that

$$1/\mu = aX_1 + bX_2 + cX_3 + \ldots + iX_i, \qquad (5\text{-}20)$$

where a, b, etc., are the values of $(1+z_i)/A_i$ and the values of z_i are to be found from the Saha equation and $X_1+X_2+ .. +X_i =1$.

Example: Consider a neutral gas made of 90% H atoms and 10% Helium atoms. We may then take the total mass of a sample of the gas to be:

$$\Sigma N_i m_i = 9m_H + 1(4m_H) = 13m_H.$$

Then $X = N_H m_H / \Sigma N_i m_i = 9m_H/ 13m_H = 0.69$ and $Y = 4m_H/ 13m_H = 0.31$. Then

$$\frac{1}{\mu} = X + \frac{1}{4}Y = 0.69 + 0.25(0.31) = 0.77$$

or $\mu = 1.30$. If the gas is completely ionized

$$\mu_{ion} = (2 \times 0.69 + 0.75 \times 0.31)^{-1} = (1.61)^{-1} = 0.62$$

Furthermore, neglecting the electrons, the number density of a gas is given by

$$n = [X + Y/A_2 + \Sigma_i(Z_i/A_i)]\rho \qquad (5\text{-}21)$$

5-4. Energy Transport

At position r within a star, the Luminosity is

$$L(r) = 4\pi r^2(F_{rad} + F_{cond} + F_{conv}) \qquad (5\text{-}22)$$

This is the total power leaving a sphere of radius r due to radiation, conduction, and convection. Let $\varepsilon(r)$ be the energy generated per unit mass at position r. This then contributes to an increment in the luminosity of

$$dL = 4\pi r^2\ \varepsilon(r)\rho(r)dr \qquad (5\text{-}23)$$

If $\varepsilon=0$ in a layer, then L is the same entering at leaving the layer. We can neglect F_{cond}, except when the density is very high, such as in a white dwarf. Energy transport by convection will occur if the density is sufficiently high or the opacity is high. We shall address convective transport later and we now turn our attention to radiative transport.

5-4A. Radiative Transport of Energy

Assume the "gray atmosphere" condition is valid, that is, the Eddington approximation for solving the transfer equation may be used. Then

$$T^4 = 3/4[T_e^4(\tau + 2/3)] \qquad (5\text{-}24)$$

and $d\tau = -\bar{\kappa}\rho dr$, where $\bar{\kappa}$ is the mean opacity at r. Now take the derivative of (5-24) with respect to optical depth:

$$\frac{d}{d\tau}(T^4) = \frac{d}{d\tau}\left\{\frac{3}{4}\left[T_e^4(\tau + 2/3)\right]\right\} \qquad (5\text{-}25)$$

$$\frac{d}{d\tau}(T^4) = \frac{3}{4}\left[T_e^4 \frac{d}{d\tau}(\tau + 2/3)\right], \qquad (5\text{-}26)$$

Since the effective temperature is not a function of the optical depth, this becomes:

$$\frac{d}{d\tau}(T^4) = \frac{3}{4}T_e^4 = 3/4F_{rad}/\sigma \qquad (5\text{–}27)$$

Change variables from τ to $\bar{\kappa}\rho$, then $d\tau = \bar{\kappa}\rho\, dr$, and we get

$$\frac{d}{d\tau}(T^4) = \frac{3}{4}T_e^4 = \frac{3}{4}\frac{F_{rad}}{\sigma} = \frac{1}{\bar{\kappa}\rho}\frac{dT^4}{dr} \tag{5-28}$$

Or, solving for the radiative flux:

$$F_{rad} = \frac{4}{3}\sigma\frac{1}{\bar{\kappa}\rho}\frac{dT^4}{dr} = \frac{4}{3}\sigma\frac{1}{\bar{\kappa}\rho}4T^3\frac{dT}{dr} \tag{5-29}$$

$$F_{rad} = \frac{16\sigma}{3}\frac{T^3}{\bar{\kappa}\rho}\frac{dT}{dr} \tag{5-30}$$

Hence the luminosity becomes

$$L(r) = 4\pi r^2 F_{rad} = 4\pi r^2\left[\frac{16\sigma}{3}\frac{T^3}{\bar{\kappa}\rho}\frac{dT}{dr}\right]$$

$$L(r) = \frac{64\pi r^2\sigma}{3}\frac{T^3}{\bar{\kappa}\rho}\frac{dT}{dr} \tag{5-31}$$

Hence, the radiative contribution to the luminosity is dependent on the opacity and the temperature gradient dT/dr. One generally thinks that the luminosity is determined by the energy generation mechanism via ε. But this is not so in the upper layers where ε is zero. After a long time, the star adjusts itself so that dT/dr is such that the energy generated is carried off at the same rate it is generated. If the mean opacity, $\bar{\kappa}$, is large, then dT/dr is steep and vice-versa.

5-4B. Convective Transport

The internal energies and densities of stars are so great that even slight mass motions have a large influence on the luminosity. Therefore, an exact theory of convection is not needed. Convection is so efficient that we don't need to ask how much energy is transported by this mechanism but only, does convection occur. If it does then F_{conv} dominates. If it does not, then F_{rad} dominates.

So we need a condition for convection to occur. Suppose a small mass element, dm, is in equilibrium for some P, T, and ρ. Consider random perturbations occurring so that dm suddenly undergoes a displacement downwards by δr. In its new surroundings the pressure is $P+\delta P$ and the density is $\rho+\delta\rho$. The excess pressure will cause contraction of the volume of the mass element until its pressure matches that of its surroundings. If this happens sufficiently fast, this will be an adiabatic process. The new density will then be

$$\rho' = \rho + d\rho = \rho + (d\rho/dP)_{ad}\delta P \tag{5-32}$$

Hence, if the mass element now has a density that is still less than its surroundings, a buoyant force will act on the element and cause it to rise back to its original level. In this case, random motions will cancel and convection will be damped. But if the element feels a force which tends to move it farther from its initial position, random motions are enhanced thereby causing convection to occur. Hence, whether or not convection occurs depend on the density-pressure gradient, $d\rho/dP$. Therefore, convection will occur if

$$(d\rho/dP)_{ad}\delta P > \delta\rho_{act} \tag{5-33}$$

Since, δP and $\delta\rho$ are very small, convection occurs if

$$\left(\frac{d\rho}{dP}\right)_{act} < \left(\frac{d\rho}{dP}\right)_{adi} \tag{5-34}$$

In essence, if density does not increase with depth fast enough, the lower layers will not be able to support the upper layers without becoming unstable to convective motions. It is more common to express the condition for convective transport in terms of T rather than ρ. From the ideal gas law, with ρ=nm,

$$\rho = mP/kT \tag{5-35}$$

Then, whenever the density, temperature, and pressure of an ideal gas vary, these changes must be related by

$$d\rho = \frac{1}{k(T/m)}\left[dP - \frac{P}{(T/m)}d\left(\frac{T}{m}\right)\right] \tag{5-36}$$

The above equation is the total differential of $d\rho$. Substitute this expression for $d\rho$ into (5-34). The minus sign then reverse the inequality so that the condition for convection becomes:

$$\left(\frac{d(T/m)}{dP}\right)_{act} > \left(\frac{d(T/m)}{dP}\right)_{adi} \tag{5-37}$$

If the average mass per free particle, m, is not changing significantly, then this simplifies to

$$\left(\frac{d(T)}{dP}\right)_{act} > \left(\frac{d(T)}{dP}\right)_{adi} \tag{5-38}$$

This says that if the temperature is changing at a faster rate than the adiabatic rate, convection will take place. The adiabatic rate can be calculated as a function of the local conditions and the composition of the gas. One may then check at any point in the model of the star to ascertain whether convection will occur from (5-38). In reality, the temperature gradient need be only slightly different than the adiabatic one for convection to carry the entire luminosity of the star from layer to layer. Hence, in those layers where convection is occurring, conditions essentially follow the adiabatic relation, viz.,

$$P = \text{constant x } \rho^{\gamma} \tag{5-39}$$,

where γ is the ratio of the specific heat at constant pressure to the specific heat at constant volume. For an ideal gas, γ = 5/3, if it is completely neutral or completely ionized. Hence in a convection zone, one need not compute the luminosity as a function of local conditions, because convection will carry whatever enters the bottom of the zone to the top of the zone.

If the mean opacity is very large, a steep temperature gradient is needed for radiation to transport the luminosity through a layer. If the opacity becomes too large, then (5-38) will be satisfied and convection occurs. This usually happens in the upper layers of cool stars where neutral H and H^- exist and result in a high opacity. The neutral hydrogen extends deeper into the star than does the H^- ion.

5-5. Constructing a Stellar Model

GIVEN: M_*, $T_*(R_*)$, R_*, $\mu(r)$, and $\rho(r)$

FIND: $M(r)$, $T(r)$, $P(r)$, $L(r)$, and $\epsilon(r)$

Gravity/Nuclear Physics $\Rightarrow$ $\epsilon(r) = \epsilon(T, \rho, \mu)$

Boundary Conditions:

At $r = 0$: $M(0) = 0$ $\quad L(0) = 0$ $\quad \rho(0) = \rho_c$ $\quad P(0) = P_c$

At $r = R_*$ $\quad M(R_*) = M_*$ $\quad L(R_*) = 4\pi R_*^2 \sigma T_*^4$

$\rho(R_*) \simeq 0$ $\quad P(R_*) \simeq 0$

Mean density: $$\bar{\rho}_* = M_* / \tfrac{4}{3}\pi R_*^3 \qquad \textbf{(5-40)}$$

Assume $\bar{\rho}_*$ exists at R_*/e,

and $\bar{\rho}_* = \rho(R_*/e) = \rho_c / e$,

From this we can solve for the central density, ρ_c.

and $$\rho(r) = \rho_c e^{-ar} \qquad (5\text{-}41)$$

Now solve for a by using $r = \frac{R_*}{e}$ in the above equation.

To find the central pressure, we integrate the hydrostatic equilibrium equation, (5-4), from r =0 to r =R

$$\Delta P = -G\int_0^R M(r)\rho(r)r^{-2}dr \qquad (5\text{-}42)$$

Let M(r) = (4/3)$\pi r^3 \langle\rho\rangle$, where $\langle\rho\rangle$ is the average density. Then

$$\Delta P = -G(4/3)\pi < \rho > \int_0^R \rho(r)r^3 r^{-2}dr \qquad (5\text{-}43)$$

We further simplify by letting $\rho = \langle\rho\rangle$, then we get

$$\Delta P \approx -G(4/3)\pi < \rho >^2 \int_0^R r\ dr = -G(4/3)\pi < \rho >^2 (\frac{1}{2})(R^2 - 0) \qquad (5\text{-}44)$$

Multiply numerator and denominator by R to retrieve the total mass of star, M(r):

$$\Delta P = P(R) - P(0) \approx -G[(4/3)\pi R^3\langle\rho\rangle](1/R)\langle\rho\rangle(1/2) \qquad (5\text{-}45)$$

The quantity in the square brackets is the total mass of the star, M(R)= M_*. Hence, the above equation becomes:

$$P(R) - P(r = 0) \approx -GM(R) < \rho > /2R\ . \qquad (5\text{-}46)$$

But P(R) = 0 and P(r=0) is P_c. Hence,

$$P_c \approx GM_*\langle\rho\rangle/2R_* \qquad (5\text{-}47)$$

Next we can get the central temperature (5-8) with P(r) = P_c.

$$T_c = P_c\mu m_H/\rho_c k. \qquad (5\text{-}48)$$

We now integrate outwards from r = 0 to r = R_*. Let r_1 be the radius of the first shell, then:

$$M(r_1) = 4\pi\int_0^{r_1} r^2\rho(r)\,dr \qquad \text{from (5-5)}$$

Then

$$\Delta P = P_c - P(r_1) = -GM(r_1)\,\rho(r_1)/r_1^2 \qquad \text{from (5-4)}$$

$$T(r_1) = P(r_1)\,\mu(r_1)\,m_H/\rho(r_1)\,k \qquad \text{from (5-8)}$$

$$L_1 = 4\pi\, r_1^2\,\epsilon(r_1)\rho(r_1)\; r_1 \qquad \text{from (5-23).}$$

In (5-23), r_1 is really dr. One then reiterates to the surface of the star. Design a program that does this.

A model consists of determining M(r), P(r), μ(r), T(r), ε(r), and L(r). In the case where $\varepsilon = 0$, L is constant. The difficult part of this endeavor is to find an expression for ρ(r). As a first approximation, one may use (5-41), but then the constant a may have to be adjusted. One might also try:

$$\rho(r) = \rho_c(1-r/R)^n. \qquad (5\text{-}49)$$

A value of n=3 is good.

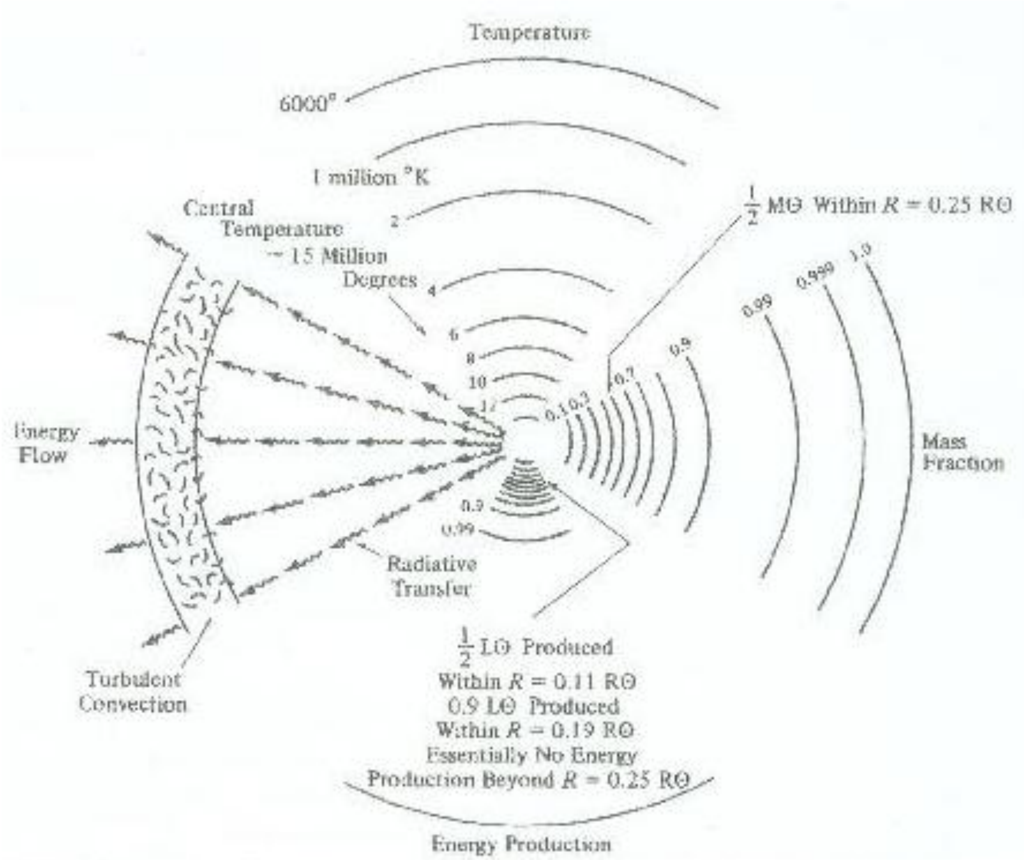
The best models of the Sun indicate:

1. Most of the H in the core has already been converted to He.

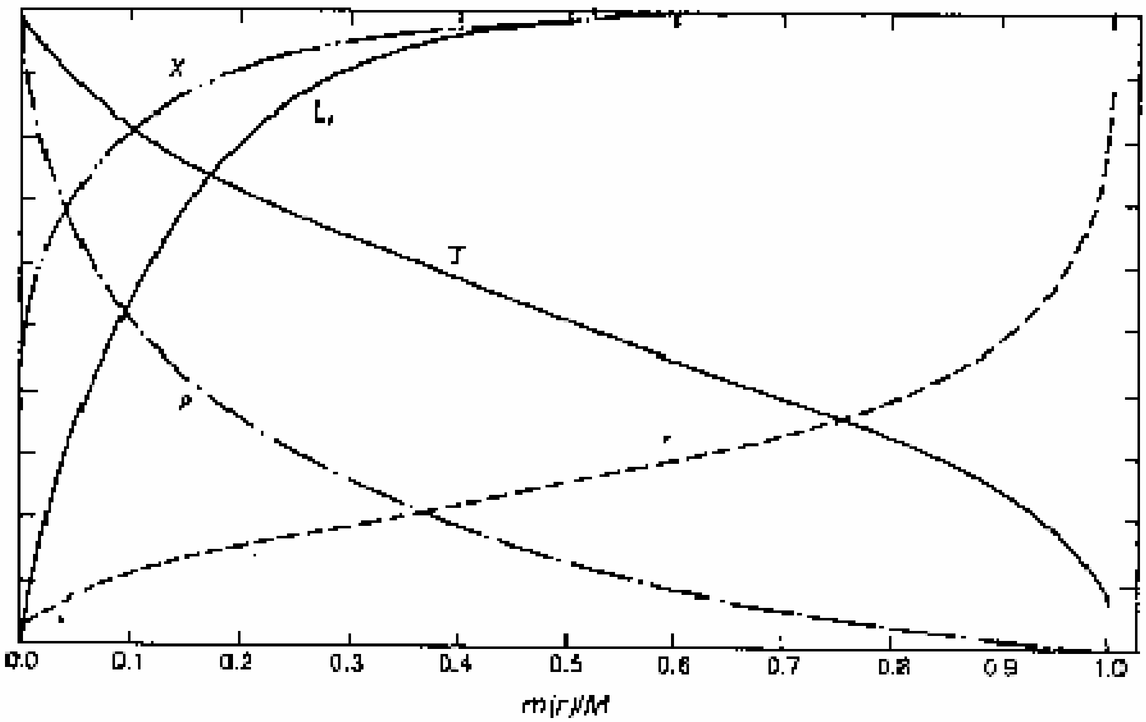
2. The p-p chain dominates all the TNF reaction in the core

3. Energy transport is radiative out to 0.8 of the Sun's radius, and then convective from there to the surface. This is because the high degree of the ionization in the interior results in a small value for the opacity, κ.

4. Convection dominates for the outer $0.2R_\odot$ because of the steep temperature gradient, dT/dr. Hence, dT/dr > adiabatic rate of cooling. Most of the opacity is due to the H^- ion.



Some physical properties of the solar interior are shown schematically (based on calculations by Iben).



A 1 $M_\odot$ model during main-sequence hydrogen burning at time 4.2699×10^9 years (between points 1 and 2 in Figure 9.1), showing radius, density, temperature, total luminosity, and hydrogen abundance versus mass fraction. The lower limits of the ordinate are zero. The upper limit for each curve is: $r = 0.9683\ R_\odot$, $\rho_c = 159.93$ g cm^{-3}, $T_c = 1.591 \times 10^7$ K, $L = 1.0575\ L_\odot$, and $X_s = 0.708$; $P_c = 2.5186 \times 10^{17}$ dynes cm^{-2}. The elapsed time is measured from the initial model for the phase before the main sequence.

Cooler stars (<5800K) have deeper convective zones and hotter stars have convective cores and no convective envelopes. This is energy generation occurs in a much smaller core and this makes the temperature gradient very steep.

V-6 Energy Sources

A. $\epsilon(r)$ is the rate of energy production per unit mass per sec.

$\epsilon(0) = 0$ except in core for M-S star

$\epsilon(r) = f(\mu, T)$ μ determines ϵ & K

$$\bar{\epsilon}_\odot = L_\odot / M_\odot = 2.0 \text{ erg/gm/sec}$$

In the core, a shell adds an increment, ΔL, to the luminosity, $L(r)$, entering the bottom of that shell

$$\Delta L = 4\pi r^2 \rho(r)\, \epsilon(r)\, \Delta r$$

B. Gravitational Contraction

$$GPE = \text{Grav. Pot. En} = -\frac{GMm}{r}$$

As a star contracts

$$GPE \longrightarrow \frac{1}{2}(\text{thermal energy}) + \frac{1}{2}\text{ Radiation.}$$

This is Kelvin-Helmholtz contraction

The availabe energy for radiation in the Sun is

$$U = \frac{1}{2} PE = \frac{GM_\odot}{2R_\odot} = 9.54 \times 10^{14} \text{ erg/gm.}$$

Now $\epsilon = U/t$ or $t = \frac{U}{\epsilon} = \frac{9.54 \times 10^{14} \text{ erg/gm}}{2.0 \times 10^{1} \text{ erg/gm/sec}}$

$t = 4.77 \times 10^{14}$ sec

or $t = 1.52 \times 10^{7}$ years. Too short to explain Main Sequence lifetime of Sun.

The Sun is older than the planets which have been dated at 4.55 billion years.

C. thermonuclear Reactions Energetics & Rates

1938 Fusion invoked to explain energy production in stars.

TNF p-p chain requires $T \simeq 5\times10^6\,K$

$$4\,{}_1H^1 \xrightarrow{TNF} {}_2He^4$$

Mass deficit for TNF:

$$4\,{}_1H^1 = 4.0312 \text{ amu } (m_H = 1.0078)$$

$${}_2He^4 = -\underline{4.0026} \text{ amu}$$

$$0.0286 \text{ mass deficit in amu}$$

From Einstein, $E = mc^2$

$$E = (0.0286)\times(1.66\times10^{-24}\,g/amu)\times9\times10^{20}\,\frac{cm^2}{sec^2}$$

$$Q = E = 4.3\times10^{-5}\,erg/\text{reaction}$$

Total Energy store of Sun, E_{Tot}

$$\frac{\text{mass deficit}}{\text{mass of 4 pt}} = \frac{.0286}{4.0312} = 0.0071$$

When this is multiplied by the original H mass of the Sun that is in the core we get E_{Tot}

about $0.1\,M_\odot$ is in the core. So

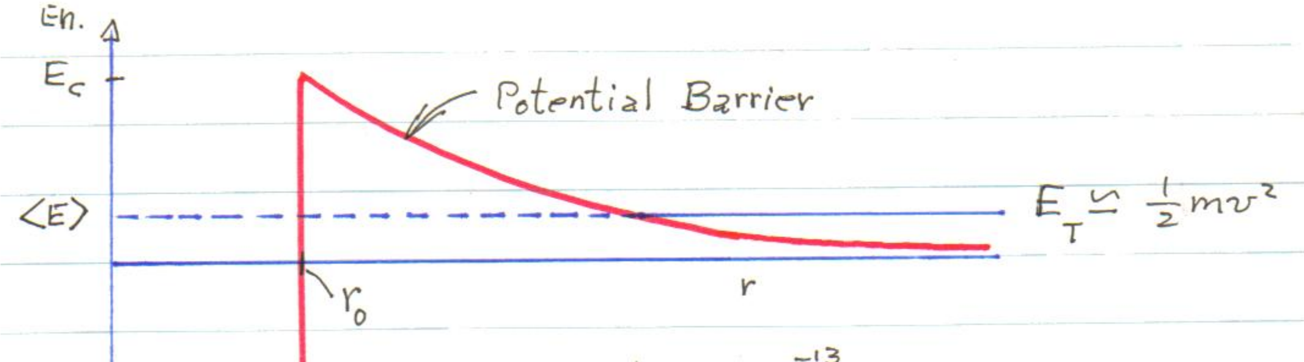
$$E_{Tot} = 0.1\,M_\odot \times c^2 \times 0.0071$$

$$E_{Tot} = 1.28\times10^{51}\,ergs$$

The value of $L_\odot = 3.90\times10^{33}$ erg/sec. So total M-S lifetime of Sun, $t_{MS} = E_{Tot}/L_\odot = 1.28\times10^{51}/3.90\times10^{33}$

$$t_{MS} = 3.28\times10^{17}\,sec = 10\times10^9 \text{ years}.$$

Nuclear Fusion Reaction Rates



$r_0 = 10^{-13}$ cm

Coulombic Barrier, $E_c \sim$ MeV $\simeq \dfrac{Z_1 Z_2 e^2}{r_n}$

Zs are the atomic nos. of the colliding nucleons

In the Sun's core $T \simeq 20 \times 10^6$ K

Protons obey Maxwellian velocity distribution law

If $f(E)$ is the fraction of the N atoms that have energy between E and E+dE, then

$$f(E)\,dE = \frac{2}{\sqrt{\pi}} \left(\frac{1}{kT}\right)^{3/2} e^{-E/kT} E^{1/2}\,dE$$

Ave KE per particle $= \langle E \rangle = \frac{3}{2} kT = \frac{1}{2} m v^2$

For $T = 20 \times 10^6$ K : $\langle E \rangle \simeq$ Kev range or $10^{-3} E_c$. So on basis of classical physics no TNF should occur. However, quantum theory permits penetration of the coulombic barrier by "tunneling". However, this depends on T also

Ignition Temperatures on basis of QM tunneling:

H fusion	5×10^6 K
He "	100×10^6 K
C "	400 to 700 $\times 10^6$ K
O "	$1.4 \to 2.0 \times 10^9$ K
Si "	$3 \to 5 \times 10^9$ K

TNF energy generation rate

$$\epsilon = \epsilon_0 \rho^a T^n \quad erg/gm/sec$$ 5-50

a and n are slowly changing functions of ρ & T.

Take $a \simeq 1$ & $n = 4$ for H fusion

$a \simeq 1$ & $n = 30$ for C fusion

ε_0 = 1.75E+6 for p-p chain and ε_0 = 7.86E+27 for CN cycle. (1)

Energy released primarily in 2 ways:

1. γ-rays
2. ν^0, neutrinoes

ν^0 are lost to star

PP-chain ($< 10^7$ K but $> 5 \times 10^6$ K)

a. $p + p \to D + e^+ + \nu_e^0 + \gamma$ 5–51

ν_e^0 is neutrino involved for electron-positron formation $\simeq$ 0.26 Mev

γ = 1.18 Mev, including positron and electron annihilation.

(1) Motz, p. 344.

b $p + D \rightarrow He^3 + \gamma$ (5.49 Mev)

c $He^3 + He^3 \rightarrow He^4 + 2p^+ + \gamma$ (12.86 Mev)

for $T > 10^7 K$ $He^3 + He^4 \rightarrow Be^7 + \gamma$ (1.59 Mev)

Steps a & b must occur twice before step c can.

step a has smaller cross section than c.

Net Q value $2(a+b)+c = 2(1.18 + 5.49) + 12.86$

$$Q_{pp} = 26.2 \text{ Mev/reaction}$$

This does not include ν^0 loss.

Rate $r = n_a n_x v \sigma(v)$ = no. of reactions per /vol/time (5-52)

where n_a = no. density of projectile particle

n_x = " " " target "

v = rel. speed of projectile in frame of target

$\sigma(v)$ = cross section for reaction in cm^2

Projectiles have range in speeds $f(v)$, so integrate over this range.

$$r = n_a n_x \int v\sigma f(v)\, dv = n_a n_x \langle \sigma v \rangle \quad (5\text{-}53)$$

So $\epsilon = \dfrac{Qr}{\rho} = \dfrac{Q n_a n_x}{\rho} \langle \sigma v \rangle = \left\{ \begin{array}{l} \text{Total energy gen.} \\ \text{per gm/sec} \end{array} \right.$ (5-54)

The rate at which the target particles are changing is the same as r given above but is a decrease.
So

$$\frac{\partial n_x}{\partial t} = -n_a n_x \langle \sigma v \rangle = -r \quad (5\text{-}55)$$

That is n_x is decreasing with time, as indicated by neg. sign.

The luminosity of a star is then:

$$L = X \int_{r=0}^{r=R_*} \varepsilon \, dm, \qquad (5\text{-}56)$$

where ε is given by (5-70) or (5-73), whichever is known. Do #115

Find the characteristic time for decrease of n_x, τ

$$\frac{dn_x}{n_x} = -n_a\langle\sigma v\rangle dt$$

$$\ln n_x = -n_a\langle\sigma v\rangle \Delta t$$

at $t=0$ $\quad n_x = n_x(0)$

So $\quad n_x = n_x(0)e^{-n_a\langle\sigma v\rangle t}$

when $n_x = n_x(0)\frac{1}{e}$

$$t = \tau = [n_a\langle\sigma v\rangle]^{-1}$$

this gives evolution time for slowest reaction, which is a. Depending on model for Sun for ρ and T in core

$$\tau_a = 7 \rightarrow 10 \times 10^9 \text{ yrs.}$$