CHAPTER 1

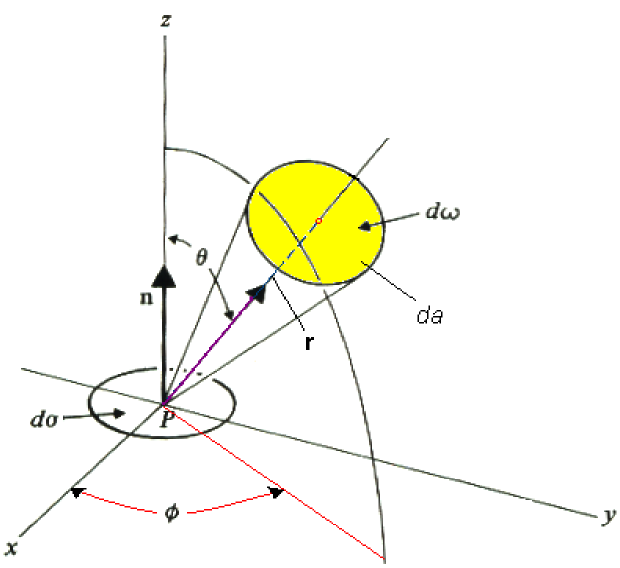
THE OBSERVATIONAL PROPERTIES OF STARS

1-1. Photometry

Photometry involves the measurement of the properties of light or electromagnetic radiation in general. More specifically, it deals with measuring the brightness of a light source, whether it be a star, planet, or galaxy. We start by defining some terms:

1-1A. Monochromatic or Specific Intensity, I_λ

$I_\lambda(\theta, \phi, t)$ is the amount of radiant energy passing through unit area per unit time, per unit wavelength, per unit solid angle. To visualize and define intensity, consider an element of area $d\sigma$, that is located at a point P in space. See the diagram to right. This element could be on the surface of a star or it could be a distance, d, from a point source of radiation, such as a star, not shown in the diagram. Let the normal of the element be along the unit vector, **n**, in the z-direction. Using spherical coordinates, imagine a beam of radiation passing through $d\sigma$ in the θ, ϕ direction within the solid angle $d\omega$ as indicated by the other vector in Fig. 1. Let da be an element of area, cut by the cone of the solid angle $d\omega$, on a spherical surface with radius r, centered on the element $d\sigma$. By definition $d\omega = da/r^2$.



Now also define $dE_\lambda(\theta, \phi, t)$ to be the radiant energy at wavelength λ that passes through $d\sigma$ per unit time, in the θ, ϕ direction. That is, $dE_\lambda(\theta, \phi, t)$ is the differential monochromatic power passing through $d\sigma$ in the θ,ϕ direction and contained within the solid angle $d\omega$. Now $dE_\lambda(\theta, \phi, t)$ is proportional to $I_\lambda(\theta, \phi, t)$ and $\cos(\theta)\, d\sigma\, d\omega$. Hence:

$$dE_\lambda(\theta, \phi, t) = I_\lambda(\theta, \phi, t) \cos\theta \, d\sigma \, d\omega \qquad (1\text{–}1)$$

This equation also defines $I_\lambda(\theta, \phi, t)$, which is also known as the specific intensity of the monochromatic radiation at wavelength λ, in the θ, ϕ, direction. The factor $\cos(\theta)\, d\sigma$ (Lambert's Law) is actually the projected area of $d\sigma$ into the θ, ϕ direction.

In spherical coordinates,

$$da = (r\sin\theta d\phi)(r d\theta),$$

so,

$$d\omega = da/r^2 = (r^2\sin\theta \; d\phi d\theta)/r^2 = \sin\theta \; d\phi d\theta$$

Then (1-1) becomes:

$$dE_\lambda(\theta, \phi, t) \;=\; I_\lambda(\theta, \phi, t)\cos\theta\;\sin\theta\;\; d\sigma\;\; d\phi d\theta d\lambda \qquad (1\text{–}2)$$

The system of units used by astrophysicists is the cgs system, not the SI system. For $I_\lambda(\theta, \phi, t)$ then, the units are ergs/cm^2/sterad/sec/Å, where 1 Angstrom is 10^{-8} cm. Nanometers (nm) are also used; 1 nm equals 10^{-7} cm. If $I_\lambda(\theta, \phi, t)$ is independent of direction, the radiation field is said to be ***isotropic***. In general, $I_\lambda(\theta, \phi, t)$ depends not only on direction but on the position of measurement and time. If $I_\lambda(\theta, \phi, t)$ is the same at all points of observation, the radiation field is said to be ***homogeneous***.

Integration of (1-2) over a sphere centered on $d\sigma$ yields the total monochromatic power passing through $d\sigma$. However, if the radiation field is isotropic, integrating (1-2) over all directions yields zero. However, for an element of area on the surface of a star, $I_\lambda(\theta, \phi, t)$ is greater on one side of $d\sigma$ than on the other and hence, there is a net flow of radiant energy outwards from the surface. In addition, integrating $I_\lambda(\theta, \phi, t)$ over a range of wavelengths (***bandpass***), $\Delta\lambda$, yields the ***polychromatic*** intensity passing through $d\sigma$ per second into a given solid angle. In order to do the latter, one must know how I_λ depends on wavelength. Do RJP-0.5

1-1B. Integrated Intensity

The quantity called the "integrated intensity" in the θ, ϕ, direction, $I(\theta, \phi, t)$, is the integral of $I_\lambda(\theta, \phi, t)$ over all wavelengths from 0 to infinity.

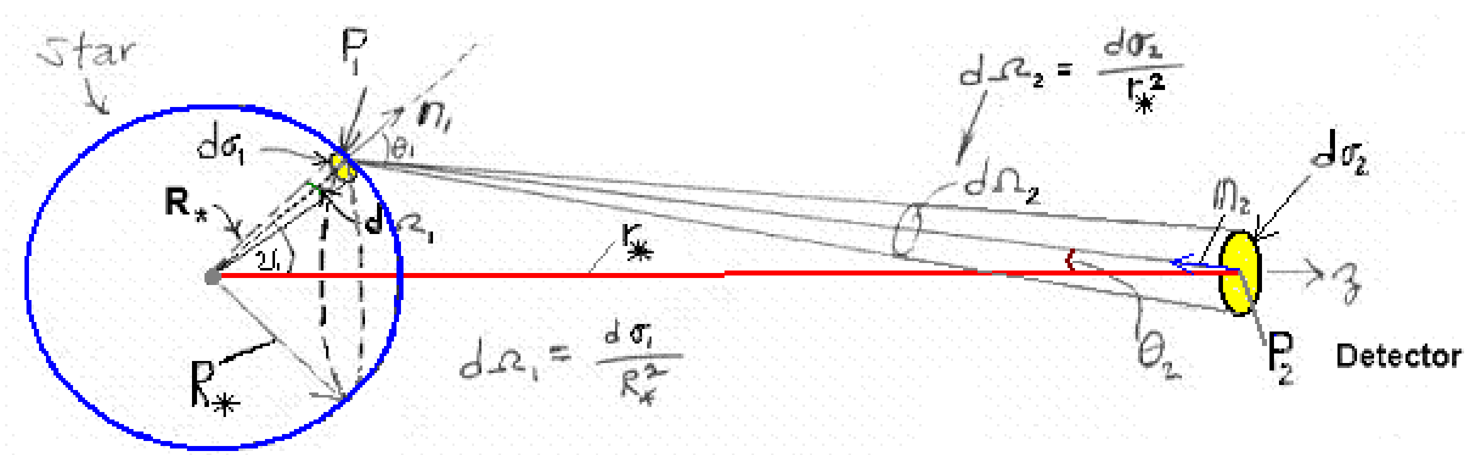
$$I(\theta, \phi, t) = \int_0^\infty I_\lambda\,(\theta, \phi, t)\, d\lambda \qquad (1\text{–}3)$$

Again, this requires knowing I_λ as a function of λ. What this function is shall be dealt with later. The integrated intensity may or may not be isotropic and/or homogeneous.

1-1C. Monochromatic Flux, F_λ

Monochromatic flux is the amount of radiant energy emitted from, passing through, or falling (incident) on a unit area per second at a given wavelength. We now derive an expression for the flux incident on a detector located at some point, P_2, in space at a distance r from the center of a spherical body such as a star.

First, consider the radiant energy from an element of the star's surface, $d\sigma_1$, located at position P_1, that falls on an element of surface $d\sigma_2$ of a detector located at P_2. See the diagram below.



Take the z-axis to be along the line from the center of the star to the surface element of the detector. The element of area, $d\sigma_1$, on the star has co-latitude υ_1 and the element of area of the detector, $d\sigma_2$, subtends solid angle $d\omega_2$ as seen from $d\sigma_1$. Hence, the amount of radiant energy emanating from $d\sigma_1$ per second at a given wavelength into solid angle $d\omega_2$ that falls on $d\sigma_2$ is given by (1-2) to be:

$$dE_\lambda(\theta, \phi, t), = I_\lambda(\theta_1, \phi_1, t)\cos(\theta_1)d\sigma_1 d\omega_2 \qquad (1\text{-}4)$$

where $I_\lambda(\theta, \phi, t)$ is the monochromatic intensity at P_1, and $\cos(\theta_1)d\sigma_1$ is the projected area of $d\sigma_1$ as seen from P_2 . For what follows, we suppress the dependence on time. Now $d\omega_2 = \cos(\theta_2)d\sigma_2 / r_*^2$. Hence,

$$dE_\lambda(\theta, \phi) = I_\lambda(\theta_1, \phi_1)\cos(\theta_1)d\sigma_1 \cos(\theta_2)d\sigma_2 / r_*^2 \qquad (1\text{-}5)$$

Now assume that $r_* >> R_*$, then $\theta_2 \approx 0$ and $\theta_1 = \upsilon_1$ for every $d\sigma$ on the star's surface. Since $\cos(\theta_2) = 1$, we get

$$dE_\lambda(\theta, \phi) = I_\lambda(\theta_1, \phi_1)\cos(\upsilon_1)d\sigma_1\, d\sigma_2 / r_*^2 \qquad (1\text{-}6)$$

Now let $d\sigma_2$ be a unit area and divide both sides of (1-6) by this to get:

$$dF_\lambda(\theta, \phi) = dE_\lambda(\theta, \phi, t)/d\sigma_2 = I_\lambda(\theta_1, \phi_1)\cos(\upsilon_1)d\sigma_1 / r_*^2 \qquad (1\text{-}7)$$

This defines the differential monochromatic flux, dF_λ, incident on a unit area of the detector from a differential surface area element of the star.

Now $d\sigma_1 = R_*^2\, d\omega_1 = R_*^2 \sin\upsilon_1 d\upsilon_1 d\phi_1$, so (1–7) becomes:

$$dF_\lambda(\theta, \phi, t) = (R_*/ r_*)^2 I_\lambda (\theta, \phi, t)\cos(\theta_1)\sin(\upsilon_1)d\upsilon_1 d\phi_1$$

But $\theta_1 \approx \upsilon_1$ when $r_* >> R_*$, hence:

$$dF_\lambda(\theta, \phi, t) = (R_*/ r_*)^2 I_\lambda (\theta, \phi, t)\cos(\upsilon_1) \sin(\upsilon_1)d\upsilon_1 d\phi_1 \qquad (1\text{-}8)$$

Now we integrate dF_λ over the hemisphere of the star facing towards the observer at P_2 to get the contributions from all the surface elements of the star emitting radiation towards the detector. We therefore can drop the subscript 1, which refers to a specific element of area on the star. The result is the flux incident on the detector.

$$\int_\sigma dF_\lambda(\theta, \phi) = \int_{\phi=0}^{\phi=2\pi} \int_{\upsilon=0}^{\upsilon=\pi/2} (R_*/ r_*)^2 I_\lambda(\theta, \phi, t) \cos(\upsilon) \sin(\upsilon)d\upsilon d\phi$$

If we assume I_λ is isotropic for stars, then

$$F_\lambda = (2\pi R_*^2 I_\lambda / r_*^2) \int_{\upsilon=0}^{\upsilon=\pi/2} \cos(\upsilon) \sin(\upsilon)d\upsilon$$

$$F_\lambda = (2\pi R_*^2 I_\lambda / r_*^2)\left[(1/2)\sin^2\upsilon\right]_0^{\pi/2} = (2\pi R_*^2 I_\lambda / r_*^2)(1/2)$$

$$F_\lambda = \pi(R_*/ r_*)^2\, I_\lambda \text{ (ergs/cm}^2\text{/sec/Å)} \qquad (1\text{-}9)$$

At the surface of the star, $r_* = R_*$ and we have

$$F^*(\lambda) = \pi\, I_\lambda \qquad (1\text{-}10)$$

Integrating over all wavelengths yields was is called the ***bolometric*** flux, F, or bolometric surface brightness of the star. Remember we have suppressed any time dependency for (1-10).

Now consider measuring the flux at two different distances from a star, r_1 and r_2. Then from (1-9) we have

$$F_\lambda (r_1)/ F_\lambda (r_2) = \pi(R_*/ r_1)^2\, I_\lambda \,/\, \pi(R_*/ r_2)^2\, I_\lambda = (r_2/r_1)^2 \qquad (1\text{-}10B)$$

This is the inverse square law for light.

Do RJP-0.6

1-1 D Brightness, b

We define measured brightness, b, to be the amount of *radiant energy* received from a light source per second in a specified portion of the spectrum, $\Delta\lambda$, or bandpass. A bandpass is an interval of contiguous wavelengths of the total electromagnetic spectrum. Brightness is actually power and may be expressed in watts or ergs per second, since astrophysicists commonly use cgs units. By *radiant energy* is meant the energy carried by electromagnetic radiation or waves. Brightness is also flux times area. See equation (1-11) below.

The brightness at a unique wavelength is called the monochromatic brightness, b_λ.

The brightness for a bandpass is a polychromatic brightness, and is the integral of the monochromatic brightness over a given interval of wavelengths, $\Delta\lambda$.

Let F_λ, the monochromatic flux, be the amount of radiant energy incident per cm^2 per second at a given wavelength (in Angstroms), arriving at the top of the Earth's atmosphere at a distance r from the source. (Motz, pgs. 11-13, uses b to represent flux).

Let S_λ be the spectral sensitivity of an instrument used for measuring brightness, such as the human eye, or a system comprised of a CCD camera and telescope. This must be determined in the laboratory.

Let k_λ be the extinction coefficient of the Earth's atmosphere, that is, the fraction of the incident flux outside the Earth's atmosphere that penetrates to the surface of the Earth. This may be determined observationally with some difficulty and depends on the line of sight through the atmosphere. (See Hiltner, W. A. in *Astronomical Techniques*, Univ. of Chicago Press, 1962)

Then the polychromatic brightness measured at the surface of the Earth is

$$\mathbf{b = \int\int k_\lambda S_\lambda F_\lambda d\lambda\, da} \tag{1-11}$$

The brightness that we have defined above is an ***apparent brightness***, that is, the brightness of an object as measured by an observer who is located a distance, r, from the object. As an example, it is the brightness as seen from the Earth. One cannot use apparent brightness to compare objects as to which are truly bright or faint.

1-1 E. Magnitudes

The determination of stellar brightness was the first quantitative measurement made for the physical properties of stars as opposed to a positional measurement. The first attempt to estimate the apparent brightnesses of the stars was made by the ancient Greek, Hipparchus, circa 150 BC. He devised what is now called the magnitude system for expressing stellar brightness. He subjectively divided all the stars visible to his unaided eye into 6 different classes of brightness. Those that he considered were the brightest were assigned the number 1, which we now call 1st magnitude or m=1. Those stars that were the faintest to be seen he classified to be 6th magnitude (m=6). The remaining stars were assigned magnitudes from 2 to 5. Note: These are apparent magnitudes because they are an attempt to measure brightness as seen from Earth. Furthermore, these are visible bandpass magnitudes.

After the application of the telescope to astronomy (Galileo Galilei, 1609), fainter stars could be seen and these have been assigned magnitudes >6. With today's technology, stars as faint as m=29 can be detected. Furthermore, the magnitude system has been defined more precisely so that fractions of a magnitude may be assigned, and more objectively, using instruments rather than the human eye.

In the modern magnitude system, a step of 5 magnitudes (Δm=5) is defined to represent a brightness ratio of exactly 100. That is, there is 100 times more light energy per second received from a first magnitude star than is received from a 6th magnitude star. Or, we receive 100 times more light energy from a 4th magnitude star than we do from a 9th magnitude star.

A step of 1 magnitude is defined to correspond to a brightness ratio equal to the 5th root of 100, or $100^{0.2}$, which is 2.512. It then follows that the brightness ratio for two stars that have any difference in magnitude, Δm, is $2.512^{\Delta m}$, or

$$b_n / b_m = 2.512^{(m-n)} = [(100)^{0.20}]^{(m-n)} = 10^{.40(m-n)}, \tag{1-12}$$

where n is the magnitude of the brighter star. Now take the log of both sides:

$$\log (b_n / b_m) = 0.4(m-n).$$

Hence,

$$m-n = 2.5\log(b_n / b_m) \tag{1-13}$$

Do problems RJP-1, 2.

For observational measurements, the magnitude of a star may be defined as:

$$m = C - 2.5\log(b), \qquad (1\text{-}14)$$

where C is a constant that characterizes the device used to measure the brightness, b. C is found by measuring b for several stars with known magnitudes, m, and then finding C by a least squares analysis. Here one must include a term that accounts for the temperature of a star. This becomes a complex problem. Of course, when two magnitudes are subtracted, as in (1-13), the constant C cancels. So for differential photometry one need not calibrate their instrument. Also, the above development regarding magnitudes is valid regardless of the bandpass. Do RJP-4.5.

One may substitute flux for what we have defined as brightness in all of the above equations without loss of meaning. (Some authors do this, e. g., Motz, Chp 1.3 and Zeilik, Chp 11.2) Do RJP-3.

Early in the 20th century, astronomers decided that a group of faint stars near the north celestial pole would anchor the zero point of the modern magnitude scale. This was done by defining the average brightness of this group to be exactly m=6.000. When this was done, some of the stars that Hipparchus had called 1st magnitude were actually brighter than 6th by more than 100 times. This necessitated introducing negative magnitudes, so that the apparent magnitude of the brightest star, Sirius, is now m = -1.47.

The magnitude system may also be assigned to any object, including the Sun (m = -26.7), Moon (m = -12.5, when full), planets (Venus gets as bright as -4.4), comets, galaxies, etc. If the Sun were viewed from the outskirts of the Solar System it would appear to have a magnitude of about -2.6 Prove this, assuming the solar system has a radius 65,000 times larger than the radius of the orbit of the Earth around the Sun. Do RJP-4.

1-1F. Combined Magnitudes:

One can not add magnitudes nor take an average when dealing with the combined light of several objects. However, one can add fluxes or brightnesses. Let b_a, and b_b, be the brightnesses of 2 stars, and let m_a and m_b be their respective magnitudes. Suppose these stars are viewed from a distance such that they are not distinguishable as separate objects (a visual double star). What would their combined magnitude be?

From (1-13):

$$m_a - m_{a+b} = 2.5 \log (b_{a+b}/b_a), \qquad (1\text{-}15)$$

where m_a is the magnitude of the brighter star and m_{a+b} is the magnitude of the combined brightnesses of stars a and b. Then:

$$m_{a+b} = m_a - 2.5 \log (b_{a+b}/b_a) = m_a - 2.5 \log [(b_a + b_b)/b_a]$$

$$m_{a+b} = m_a - 2.5 \log [1 + (b_b / b_a)]. \qquad (1\text{-}16)$$

The brightness ratio (b_b / b_a) is found using (1-12).

Example:

A double star has components of $m_a = 5.43$ and $m_b = 6.18$. What does one measure their combined magnitude to be? From (1-12),

$$(b_b / b_a) = 10^{[.4(5.43-6.18)]} = 10^{[0.4(-0.75)]} = 10^{(-0.30)} = 0.5.$$

Substitute this into (1-16) to get:

$$m_{a+b} = m_a - 2.5 \log [1 + 0.5] = 5.43 - 2.5 \log(1.5) = 5.43 - 2.5(0.18) = 5.43 - 0.45$$

or $$m_{a+b} = 4.98 .$$

The above may be generalized to any number of stars, but this becomes a very tedious calculation.

Do RJP problems 5, 6, & 7.

1-1G. Color Magnitudes

The flux emanating from the surface of an incandescent body, such as a star, has a wavelength dependence given by Planck's Law. This is shown in the diagram below for a surface temperature of 10,000 K. Therefore, one needs to specify the wavelength interval or bandpass over which the measurement of a magnitude has been made. The human eye detects what is called "visible" light or electromagnetic radiation from a wavelength of about 400 nanometers to 700 nm (4000 to 7000 Å). Hence, magnitudes measured by the eye are called "visual" magnitudes." On may construct a photometer to measure the brightness over any wavelength interval or bandpass desired.

Magnitudes that are measured for a certain defined bandpass are called color magnitudes. One of the standard color magnitude systems used by astronomers is the Johnson and Morgan U, B, V, R, I system. See the schematic below. The bandwidths for each of these color magnitudes is several decades of nanometers in the ultraviolet, blue, yellow-green, red, and near infrared portions of the EM spectrum. Note that for 10,000 K, a star is brighter in blue (smaller magnitude) than it is in the visible or red bandpasses.