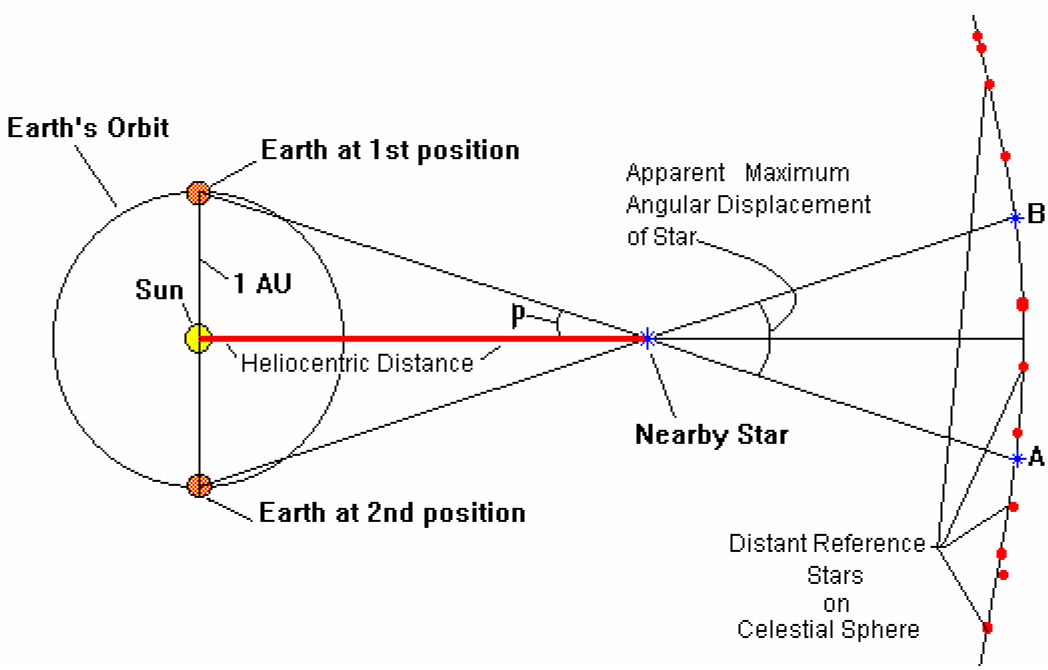
# *APPENDIX A*

## 1. Trigonometric Distances

The distance of an object may be found by measuring an angle called parallax. Parallax is defined to be the angular displacement of an object when viewed from two different positions that are located along a line perpendicular to one of the lines of sight. The idea of detecting the revolution of the Earth in its orbit around the Sun, by measuring the parallactic displacements that would result for nearby stars, goes back to Aristotle's time (about 350 BC). However, stellar parallaxes were not observed because the parallax of even the closest star is too small to be detected without a telescope. It took until 1837 AD for the first stellar parallax to be successfully measured. This was done independently by Bessel in Germany and Struve in Russia.

As the Earth moves in orbit around the Sun, a nearby star appears to change its position on the celestial sphere with respect to much more distant stars. In general, a star is observed to trace out an ellipse on the celestial sphere. The ellipse is a reflection of the Earth's orbital motion around the Sun. The eccentricity of the ellipse depends on the angle between the line of sight to the star and the direction perpendicular to the orbital plane of the Earth. The sem-imajor axis of the ellipse is the parallax. Very distant stars have negligible parallactic displacements and, therefore, serve as reference points for measuring the changing position of the star whose parallax we want to determine.

The geometry for determining the parallaxes of the stars, and, thereby, calculating their distances is illustrated in the diagram below, where the line of sight to the star lies in the plane of the Earth's orbit. In this diagram, the size of the Earth's orbit relative to the distance of the nearby star is greatly exaggerated for clarity.




When the Earth is at the 1st position in its orbit, the nearby star is seen on the celestial sphere at position A. Six months later, when the Earth has moved to the second position, the nearby star now is seen to be at position B on the celestial sphere. For any other position of the Earth in its orbit, the nearby star would be seen to have a position somewhere between these two extreme positions. Hence, in six months a star suffers its maximum angular displacement in the sky as a result of the Earth's revolution. Half of this angle is the parallax, p or $\pi$.

In the above diagram, the parallactic triangle consists of the radius of the Earth's orbit, the heliocentric distance of the star, and the angle p. Since this is a right triangle, once p is measured, the heliocentric distance of the star can be calculated using trigonometry. For the parallactic angle p shown in the above diagram, the side opposite p is the radius of the Earth's orbit, which is 1 AU. The side adjacent is the heliocentric distance of the star, D. Hence:

$$\tan p = 1 \text{ AU} / D.$$

It follows then that D = 1 AU / tan p.

Even for the closest neighboring stars of the Sun, the heliocentric distances are at least 200 thousand times greater than the radius of the Earth's orbit. Hence, parallaxes for these stars are very small fractions of a degree. A general rule is:

**The more distant the star, the smaller is its parallax.**

The sun's nearest neighbor in space is the triple star-system called Alpha Centauri. This system has a parallax of 0.750 arcseconds, which corresponds to a distance of approximately 4 light years. All other stars have parallaxes smaller than this.

Astronomers prefer to express stellar distances in parsecs (pc), where:

**1 parsec corresponds to the heliocentric distance in a parallactic triangle where p is equal to 1 arcsecond.**

The tangent of 1 arcsecond, or 0.000278 degrees, is $4.8 \times 10^{-6}$. Substituting this value into the expression D = 1 AU / tan p, we get that D = 206,265 au. This means:

**1 parsec must be 206,265 AU, or 3.26 light years.**

But remember, there is no star this close to the Sun and even Alpha Centauri is more than 1 pc away. Hence, all stars have a parallax less than 1 arcsecond.

When we express the heliocentric distance D in parsecs and p is always expressed in arcseconds, then the equation D = 1 AU / tan p simplifies to

**D = 1/p.**

For example, if a star were found to display a parallax of 0.25 arcseconds, its heliocentric distance would be D = 1/0.25 = 4 parsecs.

## 2. Stellar Motions

### **A**. Space Motion or Space Velocity

Stars are moving in orbit around the center of the Galaxy. Near the Sun, there are slight deviations of each star's motion from that of its neighbors. The difference between the Sun's motion and another star's motion is observed to be what is called a star's "space motion".

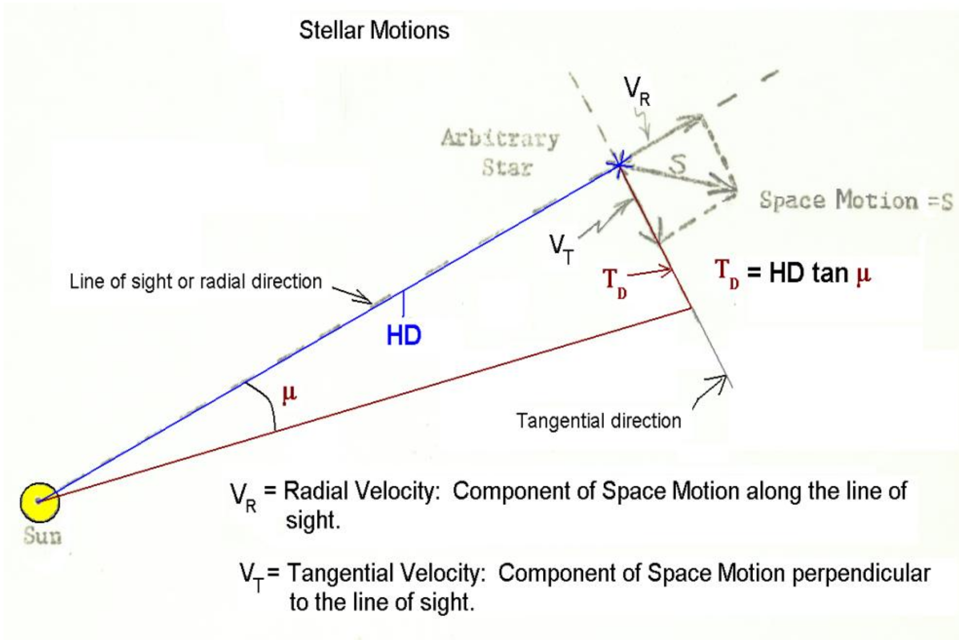
The space motion is the vector sum of what is called the tangential velocity of a star and its radial velocity. See the diagram below. The radial velocity may be determined independently of the tangential velocity by measuring the Doppler Effect in a star's spectrum. The tangential velocity may be determined if the distance of the star is known and something called the "proper motion" of the star can be measured.

### **B**. Proper Motion

**Proper motion is the annual angular rate of motion of a star across the line of sight to the star.** It is represented by the lower case Greek letter $\mu$ in the diagram, and is expressed in arcseconds per year. Proper motion is actually the projection of a star's space motion into the plane of the sky. The proper motion may be converted to the tangential velocity of the star, if the distance of the star is determined. In the diagram, the side of the triangle that is marked HD is the heliocentric distance of the star and the side labeled $T_D$ is the distance the star moves in one year traveling with velocity $V_T$. That is, the star appears to move through the angle $\mu$ in one year.

Determining the space motion of a star is complicated by the orbital motion of the Earth around the Sun and the Sun’s motion in orbit about the center the galaxy. Consider the effects of the Earth’s orbital motion, which is reflected in a star’s proper motion. Essentially the Earth’s orbital motion superposes an apparent oscillation in the star's proper motion. Once this is understood, an analysis of the observed motion of a star will yield both the distance of the star and the star's tangential velocity.

Exactly how the parallactic displacement of a star affects the observed motion of a star depends on the angular distance of the star above or below the plane of the Earth's orbit. If the star is located in the plane of the Earth's orbit and the star had no proper motion (this does not actually happen), it would appear to move back and forth along a straight line. This is the case shown in the diagram in Part **1** showing parallax.




If the star were above or below the plane of the Earth's orbit, the star would appear to move around a very small ellipse on the celestial sphere. The semi-major axis of the ellipse in arcseconds would be the parallax of the star.

The greater the angle between the plane of the Earth's orbit and the position of the star, the more circular the parallactic ellipse would look. If the star were seen in a direction that is $90^o$ above the plane of the Earth's orbit, it would be seen in the sky at the point called the **north ecliptic pole**. Then the parallactic ellipse of such a star would actually be a circle centered at the north ecliptic pole. The radius of this circle in arcseconds would be the parallax of the star.

If the star has a proper motion, this would result in the motion of the parallactic ellipse on the celestial sphere, along the direction of the proper motion.