

RJP - 531

Given

$$x = \frac{5r^2 + 5t^2}{10} - s^2$$

$$y = \frac{5s^2 - 5t^2}{10} + r^2$$

$$z = 10r^2$$

Find $J(r, s, t)$ for changing variables from x, y, z to r, s, t .

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial s} & \frac{\partial z}{\partial t} \end{vmatrix} = \begin{vmatrix} r & -2s & t \\ 2r & s & -t \\ 20r & 0 & 0 \end{vmatrix}$$

$$\begin{aligned} J &= r|0| + 2s|-20rt - 0| + t|0 + 20r^2| \\ &= -40rst + 20rst = -20rst \\ &= 20rst(1-20) \\ J &= -20rst \end{aligned}$$

The absolute value of J is what is used in the integrand.