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In a given frame of reference, a space probe has velocity $\mathbf{V} = -5\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$. when at the point $x = 10, y = 8, z = 10$. Find the component of \mathbf{V} along the direction from the above point towards the point $x = 4, y = 12, z = 2$. Find also, the unit vector along the latter direction.

Vector in direction from $10, 8, 10$ to $4, 12, 2$:

$$\mathbf{D} = (4 - 10)\mathbf{i} + (12 - 8)\mathbf{j} + (2 - 10)\mathbf{k} = -6\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}$$

Unit vector \mathbf{u}_D :

$$\mathbf{u}_D = \frac{\mathbf{D}}{D} = \frac{-6\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}}{\sqrt{6^2 + 4^2 + 8^2}} = \frac{-6\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}}{2\sqrt{29}}$$

$$\mathbf{u}_D = \frac{-3}{\sqrt{29}}\mathbf{i} + \frac{2}{\sqrt{29}}\mathbf{j} - \frac{4}{\sqrt{29}}\mathbf{k}$$

$$\mathbf{V}_D = (\mathbf{V} \cdot \mathbf{u}_D)\mathbf{u}_D = \left(\frac{15}{\sqrt{29}} + \frac{16}{\sqrt{29}} - \frac{16}{\sqrt{29}} \right) \mathbf{u}_D = \frac{15}{\sqrt{29}} \mathbf{u}_D$$

$$\mathbf{V}_D = \frac{-45}{29}\mathbf{i} + \frac{30}{29}\mathbf{j} - \frac{60}{29}\mathbf{k}$$

$$\mathbf{V}_D = -1.55\mathbf{i} + 1.03\mathbf{j} - 2.07\mathbf{k}$$