$\boxed{\text{RJP-341}}$

Given $\mathbf{V} = 7i + 9j + 4k$, find V_x' V_y' V_z' in frame rotated relative to original frame by $20°$ around z-axis. Prove invariance of $|\mathbf{V}|$.

$$V' = RV$$

$$\begin{pmatrix} V_x' \\ V_y' \\ V_z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$\begin{pmatrix} V_x' \\ V_y' \\ V_z' \end{pmatrix} = \begin{pmatrix} \cos 20° & \sin 20° & 0 \\ -\sin 20° & \cos 20° & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 7 \\ 9 \\ 4 \end{pmatrix}$$

$$V_x' = V_x \cos\theta + V_y \sin\theta + V_z \cdot 0 = 7(.94) + 9(.34) + 0$$

$$V_y' = V_x(-\sin\theta) + V_y \cos\theta + V_z \cdot 0 = -7(.34) + 9(.94) + 0$$

$$V_z' = V_x \cdot 0 + V_y \cdot 0 + V_z = 4$$

$$V_x' = 6.58 + 3.06 = 9.64$$

$$V_y' = -2.38 + 8.46 = 6.08$$

$$V_z' = 4 = 4$$

$$V'^2 = V_x'^2 + V_y'^2 + V_z'^2 = 92.93 + 36.97 + 16 = 145.90$$

$$|\mathbf{V}'| = V' = \sqrt{145.90} = 12.08$$

$$V^2 = 7^2 + 9^2 + 4^2 = 49 + 81 + 16 = 146$$

$$|\mathbf{V}| = \sqrt{146} = 12.08$$

Hence $|\mathbf{V}| = |\mathbf{V}'|$, that is, $|\mathbf{V}|$ is invariant