RJP-331 Row Reduction Method to Reduced Echelon Form:

Solve the following set of linear equations by row reduction:

$$
\begin{aligned}
2x + 3y - z &= -3 \\
x + y + z &= 2 \\
-x + y + 2z &= 2
\end{aligned}
$$

$$
\begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}
\begin{pmatrix} x \\ y \\ z \end{pmatrix}
=
\begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}
$$

Form augmented matrix

$$
\left(\begin{array}{ccc:c} 2 & 3 & -1 & -3 \\ 1 & 1 & 1 & 2 \\ -1 & 1 & 2 & 2 \end{array}\right)
$$

Our goal is to reduce these rows so that zeros occur in the boxed area. With this completed the linear equations are then simple enough to solve by back substitution. We include the fourth column because, what you do to one side of an equation you must do to the other.

$$
\begin{pmatrix} 2 & 3 & -1 & -3 \\ 1 & 1 & 1 & 2 \\ -1 & 1 & 2 & 2 \end{pmatrix}
\xrightarrow[\text{replace row 3 with } (2R_3 + R_2)]{\text{replace row 2 with } (-1)(2R_2 - R_1)}
\begin{pmatrix} 2 & 3 & -1 & -3 \\ 0 & 1 & -3 & -7 \\ 0 & 5 & 3 & 1 \end{pmatrix}
\xrightarrow{R_3 - 5R_2}
\begin{pmatrix} 2 & 3 & -1 & -3 \\ 0 & 1 & -3 & -7 \\ 0 & 0 & 18 & 36 \end{pmatrix}
\rightarrow
$$

$$
\begin{pmatrix} 2 & 3 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & 18 \end{pmatrix}
\begin{pmatrix} x \\ y \\ z \end{pmatrix}
=
\begin{pmatrix} 3 \\ -7 \\ 36 \end{pmatrix}
\implies
\begin{aligned}
&1) & 2x + 3y - z &= 3 \\
&2) & y - 3z &= -7 \\
&3) & 18z &= 36
\end{aligned}
$$

3) $18z = 36 \implies z = 2$ by back substitution

2) $z = 2$: $y - 3(z) = -7 \implies y = -1$

3) $z = 2, y = -1$: $2x + 3(-1) - 2 = -3 \implies x = 1$

$\implies x = 1$, $y = -1$, and $z = 2$