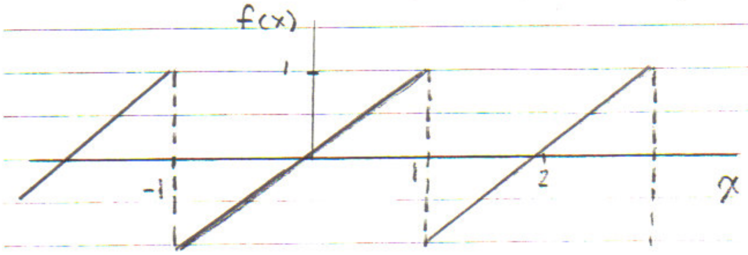
7-8.15

$f(x) = x$ on $-1 < x < 1$ $\quad 2\ell = 2 \quad \ell = 1$



$$c_n = \frac{1}{2\ell}\int_{-\ell}^{\ell} f(x)\, e^{-in\pi x/\ell}\, dx = \frac{1}{2}\int_{-1}^{1} x\, e^{-in\pi x}\, dx$$

$$c_n = \frac{1}{2}\left[\frac{e^{-in\pi x}}{(-in\pi)^2}(-in\pi x - 1)\right]_{-1}^{1} = -\frac{1}{2n^2\pi^2}\left[(-1)^n(-in\pi - 1) - (-1)^n(in\pi - 1)\right]$$

Since $e^{\pm in\theta} = \cos n\theta \pm i \sin n\theta \qquad \cos n\pi = (-1)^n \quad \sin n\pi = 0$

$$c_n = -\frac{(-1)^n}{2n^2\pi^2}\left[-in\pi - 1 - in\pi + 1\right] = \frac{-(-1)^n}{2n^2\pi^2}(-2in\pi) = \frac{i(-1)^n}{n\pi}$$

Hence $$f(x) = \sum_{-\infty}^{\infty} \frac{i(-1)^n}{n\pi} e^{in\pi x} = \frac{i}{\pi}\sum \frac{(-1)^n}{n} e^{in\pi x}$$

or $$f(x) = \frac{i}{\pi}\left[\cdots + \frac{e^{-3i\pi x}}{3} - \frac{e^{-2i\pi x}}{2} + e^{-i\pi x} - e^{+i\pi x} + \frac{e^{2i\pi x}}{2} + \cdots\right]$$