7-5.1

$$f(x) = \begin{cases} 1 & -\pi < x < 0 \\ 0 & 0 < x < \pi \end{cases}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} 1\cdot\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} 0\cdot\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\left[\frac{\sin nx}{n}\right]_{-\pi}^{0} = \frac{1}{\pi}\left[\frac{\sin 0}{n} - \frac{\overset{0}{\cancel{\sin(-n\pi)}}}{n}\right] = 0$$

$\therefore\ a_n = 0 \qquad n \neq 0$

However $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} dx = \frac{1}{\pi}(\pi - 0) = 1$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} 1\cdot\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} 0\cdot\sin nx\,dx$$

$$b_n = \frac{1}{\pi}\left[\frac{-\cos nx}{n}\right]_{-\pi}^{0} = \frac{1}{\pi}\left[\frac{-1}{n} + \frac{\cos(-n\pi)}{n}\right]$$

$$b_n = \frac{1}{n\pi}\left(-1 + (-1)^n\right) = \begin{cases} 0 & \text{even } n \\ \frac{-2}{n\pi} & \text{odd } n \end{cases}$$

Hence

$$f(x) = \frac{1}{2}a_0 - \frac{2}{\pi}\sum_{\substack{\text{odd}\\ n}} \frac{\sin(nx)}{n}$$

$$f(x) = \frac{1}{2} - \frac{2}{\pi}\left[\frac{\sin x}{1} + \frac{\sin 3x}{3} + \cdots\right]$$