7.3-9

We use the trigonometry formula

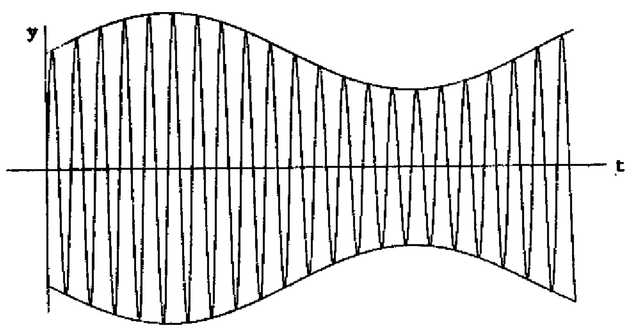
$$\sin\theta\sin\phi = \frac{1}{2}[\cos(\theta-\phi) - \cos(\theta+\phi)]$$

to write

$$y = (A + B\sin 2\pi f t)\sin 2\pi f_c\left(t - \frac{x}{v}\right) = A\sin 2\pi f_c\left(t - \frac{x}{v}\right)$$
$$+\frac{1}{2}B\left\{\cos 2\pi\left[(f - f_c)t + f_c x/v\right] - \cos 2\pi\left[(f + f_c)t - f_c x/v\right]\right\} .$$

Using the values given for A, B, f, and f_c, we sketch, at $x = 0$, the graph of

$$y = (3 + \sin 2\pi t)\sin 40\pi t.$$



7.4-5

Since $\cos^2(x/2)$ has period 4π, we see that 0 to $\pi/2$ is less than a quarter period so we cannot use the text discussion following Figure 4.2. By text equation (4.3) and integral tables, we find

$$\text{Average of } \cos^2\frac{x}{2} \text{ on } (0,\pi/2) \quad \text{is} \quad \frac{1}{\pi/2}\int_0^{\pi/2}\cos^2\frac{x}{2}\,dx$$

$$= \frac{2}{\pi}\left(\frac{1}{2}x + \frac{1}{2}\sin x\right)\Big|_0^{\pi/2} = \frac{2}{\pi}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{\pi} .$$

7.4-11

Since 2π is a period of $\sin x$, the ave. of $\sin x$ on interval 0 to 2π is 0. The ave of $\sin^2 x$ is $\frac{1}{2}$ by (4.8) thus ave. of $\sin x + \sin^2 x$ is $\frac{1}{2}$.