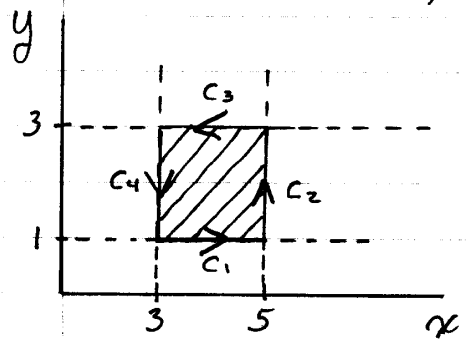
Boas 6-9.10 Example of Green's theorem in a Plane

Evaluate $\oint_C (2y\,dx - 3x\,dy)$ around a square bounded by

$$x = 3,\ x = 5,\ y = 1,\ y = 3$$



By Green's theorem:

with $P = 2y$ & $Q = -3x$

Then

$$\oint_C (2y\,dx - 3x\,dy) =$$

$$\int_{y=1}^{3} \int_{x=3}^{5} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$\frac{\partial Q}{\partial x} = -3$ $\frac{\partial P}{\partial y} = 2$, So

$$\int_y \int_x (-3-2)\,dx\,dy = -5 \iint_{yx} dx\,dy = -5(2)(2) = -20$$

Now show $\oint_C (2y\,dx - 3x\,dy)$ is the same:

$$\int_{C_1} (2 \cdot 1)\,dx + \int_{C_2} (-15)\,dy + \int_{C_3} 6\,dx + \int_{C_4} (-9)\,dy$$

$$2\left[x\right]_3^5 + (-15)\left[y\right]_1^3 + 6\left[x\right]_5^3 + (-9)\left[y\right]_3^1$$

$$2(5-3) + (-15)(3-1) + 6(3-5) + (-9)(1-3)$$

$$4 + (-30) + (-12) + 18 = -20$$

This proves the validity of Green's theorem in a plane.