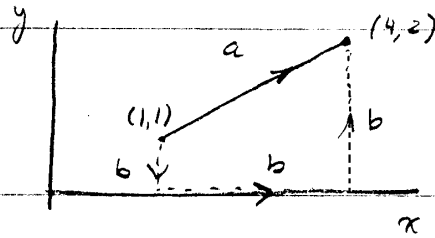


Boor 6-8.5

$$F = x^2 y^2 i - x y^2 j$$

$$\nabla \times F \neq 0$$



a)  $dW = F \cdot d\vec{r} = (x^2 y^2 i - x y^2 j) \cdot (dx i + dy j)$

$$dW = x^2 y dx - x y^2 dy$$

along a, slope  $m = 1/3$   $b = \text{intercept} = 2/3 = \frac{y_1 x_2 - y_2 x_1}{x_2 - x_1}$   
 $y = 2/3 + 1/3 x$   $dy = 1/3 dx$

$$\begin{aligned} dW &= x^2 (2/3 + 1/3 x) dx - x (2/3 + 1/3 x)^2 \frac{1}{3} dx \\ &= \frac{2}{3} x^2 dx + \frac{1}{3} x^3 dx - x (4/9 + 2 \cdot \frac{2}{9} x + \frac{1}{9} x^2) \frac{1}{3} dx \\ &= \frac{2}{3} x^2 dx + \frac{1}{3} x^3 dx - \frac{4}{27} x dx - \frac{4}{27} x^2 dx - \frac{1}{27} x^3 dx \\ &= -\frac{4}{27} x dx + \frac{14}{27} x^2 dx + \frac{8}{27} x^3 dx \end{aligned}$$

$$\begin{aligned} W &= \int dW = -\int_1^4 \frac{4}{27} x dx + \int_1^4 \frac{14}{27} x^2 dx + \int_1^4 \frac{8}{27} x^3 dx \\ &= -\frac{4}{27} \frac{x^2}{2} \Big|_1^4 + \frac{14}{27} \frac{x^3}{3} \Big|_1^4 + \frac{8}{27} \frac{x^4}{4} \Big|_1^4 \\ &= -\frac{4}{27} \frac{1}{2} (16-1) + \frac{14}{27} \frac{1}{3} (64-1) + \frac{8}{27} \frac{1}{4} (256-1) \\ &= -\frac{4 \cdot 15}{2 \cdot 27} + \frac{14 \cdot 63}{27 \cdot 3} + \frac{8 \cdot 255}{4 \cdot 27} = -\frac{30}{27} + \frac{294}{27} + \frac{510}{27} \\ &= \frac{774}{27} = 28.67 \end{aligned}$$

b) along path b:

$$\begin{aligned} dW &= F \cdot (+dy j) + F \cdot (dx i) + F \cdot (dy j) \\ &= \underbrace{-x y^2 dy}_{x=1} + \underbrace{x^2 y dx}_{y=0} - \underbrace{x y^2 dy}_{x=4} \end{aligned}$$

$$\begin{aligned} W &= \int dW = -\int_1^0 y^2 dy + 0 - 4 \int_0^2 y^2 dy = \frac{y^3}{3} \Big|_1^0 - \frac{4}{3} y^3 \Big|_0^2 \\ &= (0 - \frac{1}{3}) - \frac{4}{3} (8 - 0) = -\frac{31}{3} \end{aligned}$$