B. 6-8.18

Given force field $\mathbb{F} = -y\,\mathbb{i} + x\,\mathbb{j} + z\,\mathbb{k}$,

Find $\int \mathbb{F}\cdot d\mathbb{r} = \int(-y\,dx + x\,dy + z\,dz)$ along paths from $(1,0,0)$ to $(-1,0,\pi)$

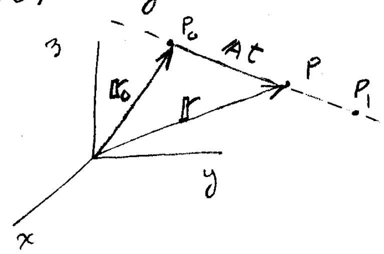
(a) along the helix, we have

$$x = \cos t \qquad dx = -\sin t\,dt$$
$$y = \sin t \qquad dy = \cos t\,dt$$
$$z = t \qquad dz = dt$$

where t goes from 0 to π. Then

$$\int \mathbb{F}\cdot d\mathbb{r} = \int_0^{\pi} (-\sin t)^2\,dt + \cos^2 t\,dt + t\,dt)$$
$$= \int_0^{\pi} (1+t)\,dt = \pi + \frac{\pi^2}{2}$$

(b) along line between pts. $\mathbb{r} = \mathbb{r}_0 + \mathbb{A}t$



Let $P_0 = 1,0,0 \qquad P_1 = -1,0,\pi$

Then $\mathbb{r}_0 = 1\cdot\mathbb{i} + 0\,\mathbb{j} + 0\cdot\mathbb{k} = \boxed{\mathbb{i}}$

$$\mathbb{A} = (x_1 - x_0)\,\mathbb{i} + (y_1 - y_0)\,\mathbb{j} + (z_1 - z_0)\,\mathbb{k}$$
$$= (-1-1)\,\mathbb{i} + (0-0)\,\mathbb{j} + (\pi - 0)\,\mathbb{k}$$
$$\mathbb{A} = (-2)\,\mathbb{i} + \pi\mathbb{k}$$
$$\mathbb{r} = \mathbb{r}_0 + \mathbb{A}t = \boxed{\mathbb{i}} + (-2\,\mathbb{i} + \pi\mathbb{k})t$$

Or $\mathbb{r} = \underbrace{(1-2t)}_{x}\,\mathbb{i} + \underbrace{(\pi t)}_{z}\,\mathbb{k} \qquad y = 0$

at $P_0 \qquad x = 1 = 1-2t \quad \& \quad z = 0 = \pi t \qquad$ so $t = 0$

at $P_1 \qquad x = -1 = 1-2t \quad \& \quad z = \pi = \pi t \qquad$ so $t = 1$

Hence $\int \mathbb{F}\cdot d\mathbb{r} = \int_0^1 \pi t\cdot\pi\,dt = \pi^2/2$

Now $\text{curl}\,\mathbb{F} = \begin{vmatrix} \mathbb{i} & \mathbb{j} & \mathbb{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & z \end{vmatrix} = 0\,\mathbb{i} - 0\,\mathbb{j} + (1+1)\mathbb{k} = 2\mathbb{k}$

$\therefore \nabla\times\mathbb{F} \neq 0$

So, work integrals should be different, as calculated.