Boas 6-7.5

$\mathbb{V} = x^2 i + y^2 j + z^2 k$

$\nabla \cdot \mathbb{V} = \partial V_x/\partial x + \partial V_y/\partial y + \partial V_z/\partial z$

$= 2x + 2y + 2z$

$$\nabla \times \mathbb{V} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & y^2 & z^2 \end{vmatrix} = (0-0)i - (0-0)j + (0-0)k$$

$\therefore \quad \nabla \times \mathbb{V} = 0$

Boas 6-7.14

$\phi = (x^2+y^2+z^2)^{-1/2}$

$\nabla^2 \phi = \partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 + \partial^2\phi/\partial z^2$

$\partial^2\phi/\partial x^2 = \frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1/2}\right] = \frac{\partial}{\partial x}\left[(-\tfrac{1}{2})(x^2+y^2+z^2)^{-3/2}(2x)\right]$

$= (-1)\frac{\partial}{\partial x}\left[(x)(x^2+y^2+z^2)^{-3/2}\right]$

$= (-1)(x^2+y^2+z^2)^{-3/2} - x\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-3/2}$

$= -(x^2+y^2+z^2)^{-3/2} - x(-3/2)(x^2+y^2+z^2)^{-5/2}(2x)$

$= -(x^2+y^2+z^2)^{-3/2} + 3x^2(x^2+y^2+z^2)^{-5/2}$

Similarly,

$\partial^2\phi/\partial y^2 = -(x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2}$

$\partial^2\phi/\partial z^2 = -(x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2}$

Adding: $\nabla^2\phi = -3(x^2+y^2+z^2)^{-3/2} + 3(x^2+y^2+z^2)(x^2+y^2+z^2)^{-5/2}$

$= -3(x^2+y^2+z^2)^{-3/2} + 3(x^2+y^2+z^2)^{-3/2} = 0$