Boas $\boxed{6\text{-}6.13}$

Let $\phi = e^x \cos y$.

(a) Most rapid rate of change of ϕ at $(1, -\pi/4)$:

$$\nabla\phi = \frac{\partial}{\partial x}(e^x \cos y)\,\hat{i} + \frac{\partial}{\partial y}(e^x \cos y)\,\hat{j}$$

$$\nabla\phi = (e^x \cos y)\hat{i} - (e^x \sin y)\hat{j}$$

at $(1, -\pi/4)$: $\nabla\phi = e^1 \cos(-\frac{\pi}{4})\hat{i} - e^1 \sin(-\frac{\pi}{4})\hat{j} = \frac{e\sqrt{2}}{2}\hat{i} + \frac{e\sqrt{2}}{2}\hat{j}$

$$|\nabla\phi| = \left[\left(e\frac{\sqrt{2}}{2}\right)^2 + \left(e\frac{\sqrt{2}}{2}\right)^2\right]^{1/2} = \left[\frac{e^2}{2} + \frac{e^2}{2}\right]^{1/2} = e$$

(b) For direction $\hat{i} + \hat{j}\sqrt{3}$: $\quad m = \frac{1}{2}(\hat{i} + \hat{j}\sqrt{3})$

$$\nabla\phi \cdot m = \left[(e^x \cos y)\hat{i} - (e^x \sin y)\hat{j}\right] \cdot \left[\frac{1}{2}(\hat{i} + \hat{j}\sqrt{3})\right]$$

$$= \frac{1}{2} e^x \cos y - \frac{\sqrt{3}}{2} e^x \sin y$$

at $(0, \pi/3)$

$$\nabla\phi \cdot m = \frac{d\phi}{ds} = \frac{1}{2} e^0 \underbrace{\cos(\tfrac{\pi}{3})}_{=\frac{1}{2}} - \frac{\sqrt{3}}{2} e^0 \underbrace{\sin(\tfrac{\pi}{3})}_{\frac{\sqrt{3}}{2}}$$

$$\frac{d\phi}{ds} = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$$

(c) $E = -\nabla\phi = (-e^x \cos y)\hat{i} + (e^x \sin y)\hat{j}$

at $(0, \pi)$ $\quad E = (-\underbrace{e^0}_{1} \underbrace{\cos\pi}_{-1})\hat{i} + e^0 \underbrace{\sin\pi}_{0})\hat{j} = \hat{i} \qquad |E| = 1$

(d) E at $x = -1$ for any y

$$E = -\left(\frac{\cos y}{e}\right)\hat{i} + \left(\frac{\sin y}{e}\right)\hat{j}$$

$$|E| = \sqrt{\frac{\cos^2 y}{e^2} + \frac{\sin^2 y}{e^2}} = \sqrt{\frac{1}{e^2}} = \frac{1}{e}$$