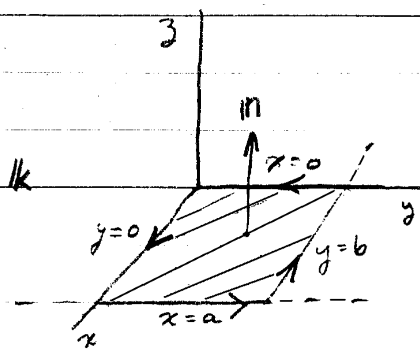
$\boxed{6\text{-}11.2}$

(a)

$$\nabla \times \mathbb{A} = \begin{vmatrix} \mathbb{i} & \mathbb{j} & \mathbb{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2-y^2 & 2xy & 0 \end{vmatrix} = 0\,\mathbb{i} - 0\,\mathbb{j} + \left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial y}(x^2-y^2)\right]\mathbb{k}$$

$$\nabla \times \mathbb{A} = (2y + 2y)\mathbb{k} = 4y\,\mathbb{k}$$




(b) $\int (\nabla \times \mathbb{A})\cdot \mathbb{n}\, d\sigma$ where $\mathbb{n} = \mathbb{k}$

$$\int (4y\,\mathbb{k})\cdot \mathbb{k}\, d\sigma = \int_0^b\int_0^a 4y\, dx\, dy$$

$$= 4a\int_0^b y\, dy = 2ab^2$$

(c) $\oint \mathbb{A}\cdot d\mathbb{r} = \int_0^a A_x\,dx\Big|_{dy=0} + \int_0^b A_y\,dy\Big|_{dx=0} + \int_a^0 A_x\,dx\Big|_{dy=0} + \int_b^0 A_y\,dy\Big|_{dx=0}$

$(d\mathbb{r} = \mathbb{i}dx + \mathbb{j}dy + \mathbb{k}dz)$

in $xy$ plane $d\mathbb{r} = \mathbb{i}dx + \mathbb{j}dy$

$$= \int_0^a (x^2-y^2)\,dx + \int_0^b 2xy\,dy + \int_a^0 (x^2-y^2)\,dx + \int_b^0 2xy\,dy$$

$$= \int_0^a (x^2-0^2)\,dx + \int_0^b 2ay\,dy + \int_a^0 (x^2-b^2)\,dx + \int_b^0 2\cdot 0\cdot y\,dy$$

$$= \int_0^a x^2\,dx + 2a\int_0^b y\,dy + \int_a^0 x^2\,dx - \int_a^0 b^2\,dx$$

$$= \frac{x^3}{3}\Big|_0^a + 2a\frac{y^2}{2}\Big|_0^b + \frac{x^3}{3}\Big|_a^0 - b^2x\Big|_a^0$$

$$= \frac{a^3}{3} + ab^2 - \frac{a^3}{3} + b^2a$$

$$\oint \mathbb{A}\cdot d\mathbb{r} = 2ab^2 \qquad \text{Q.E.D.}$$