Chap. 5

Section 4

1. (a) $A = \int_{r=0}^{a} \int_{\theta=0}^{2\pi} r \, dr \, d\theta = \int_0^a r \, dr \int_0^{2\pi} d\theta = \pi a^2$

(b) $\iint \bar{x} r \, dr \, d\theta = \iint x r \, dr \, d\theta = \int_{r=0}^{a} \int_{\theta=0}^{\pi/2} r \cos\theta \, r dr \, d\theta = a^3/3$

Since the area of one quadrant $= \frac{\pi a^2}{4}$, $\bar{x} = \frac{a^3}{3} \div \frac{\pi a^2}{4} = \frac{4a}{3\pi}$.

In a similar way, or by symmetry, $\bar{y} = \frac{4a}{3\pi}$.

(c) $I_x = \rho \iint y^2 r \, dr \, d\theta = \rho \int_{r=0}^{a} \int_{\theta=0}^{2\pi} r^2 \sin^2\theta \, r \, dr \, d\theta = \frac{\pi a^4 \rho}{4} = Ma^2/$

(See solution of Chapter 2 Problem 11.12 for an easy way to the integral of $\sin^2\theta$.)

(d) $C = \int_0^{2\pi} a \, d\theta = 2\pi a$

(e) $\int \bar{x} \, a \, d\theta = \int x a \, d\theta = \int_0^{\pi/2} a \cos\theta \, a \, d\theta = a^2$.

Since the quarter circle arc length $= \frac{\pi a}{2}$, $\bar{x} = a^2 \div \frac{\pi a}{2} = \frac{2a}{\pi}$.

Similarly, or by symmetry, $\bar{y} = 2a/\pi$.