

**Boas 5-3.33**

The integral for the volume is given by equation (3.10) on page 254.

Doing the  $y$  integral first and evaluating at the upper and lower limits for  $y$  we get:

$$V = \int_x dx \int_{z=-x^2}^{x^2} \left[ 2\sqrt{x^4 - z^2} \right] dz$$

From tables

$$V = \int_0^1 dx \cdot 2 \cdot \frac{1}{2} \left[ z\sqrt{x^4 - z^2} + x^4 \sin^{-1} \frac{z}{x^2} \right]_{-x^2}^{x^2}$$

$$V = \int_0^1 dx \left[ \left( x^2 \sqrt{x^4 - x^4} + x^4 \sin^{-1} \frac{x^2}{x^2} \right) - \left( -x^2 \sqrt{x^4 - x^4} + x^4 \sin^{-1} \frac{-x^2}{x^2} \right) \right]$$

$$= \int_0^1 dx \left[ \left( x^4 \sin^{-1} 1 \right) - \left( x^4 \sin^{-1} -1 \right) \right] = \int_0^1 dx \left[ x^4 \left( \frac{\pi}{2} \right) 2 \right]$$

$$V = \int_0^1 \pi x^4 dx \quad \text{as obtained before}$$

$$V = \frac{\pi}{5}$$