Boas 5.3-3

Thin rod $\ell = 10$ ft. ρ varies 4 to 24 lb/ft

$$\rho = 4 + \frac{20}{10}x = 4 + 2x$$

(a.) $$M = \int \rho\, dV = \int \rho\, dx = \int_0^{10} (4+2x)\,dx = 4x\Big|_0^{10} + \frac{2x^2}{2}\Big|_0^{10}$$

$$= 40 + (10^2 - 0) = 40 + 100 = 140 \text{ lb mass}$$

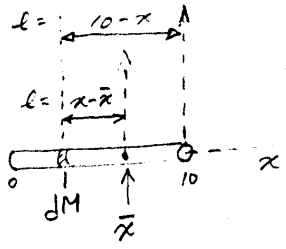
(b.) $\bar{x}$:

$$\bar{x}\int dM = \int x\rho\, dx = \int_0^{10} (4x + 2x^2)\,dx$$

$$140(\bar{x}) = \left(\frac{4}{2}x^2 + \frac{2}{3}x^3\right)_0^{10} = 2\cdot 10^2 + \frac{2}{3}(10)^3$$

$$= 200 + \frac{2}{3}(1000) = \frac{2600}{3}$$

$$\bar{x} = \frac{130}{21} = 6.19$$



(c.) $$I_{\bar{x}} = \int (x - \bar{x})^2\, dM = \int_0^{10} (x - \bar{x})^2 \rho\, dx$$

$$I = \int_0^{10} \left(x - \frac{130}{21}\right)^2 (4 + 2x)\,dx = \frac{61}{63} \times 10^3$$

$$I/M = \frac{61}{63} \times 10^3 \div 140 = 6.92$$

(d.) $$I = \int_0^{10} (10 - x)^2 (4 + 2x)\,dx$$

Multiply factors in integrand:

$$I = \int_0^{10} (100 - 20x + x^2)(4 + 2x)\,dx$$

$$I = \int_0^{10} \left[400 - 80x + 4x^2 + 200x - 40x^2 + 2x^3\right] dx$$

$$I = \int_0^{10} \left[400 + 120x - 36x^2 + 2x^3\right] dx = 3000 = \frac{3000\,M}{140} = \frac{150}{7}M$$

$$I = 21.43M$$