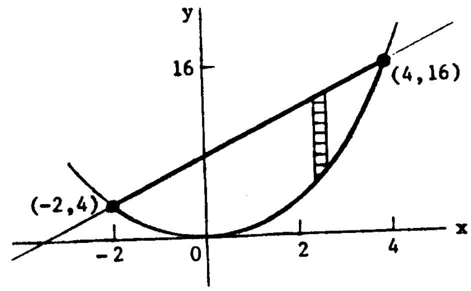
Boas 5-2.11

11. We first find the points of intersection of the line $y = 2x + 8$ and the parabola $y = x^2$.



$$x^2 = 2x + 8$$

$$x^2 - 2x - 8 = (x - 4)(x + 2) = 0$$

$$x = -2, \qquad x = 4.$$

Compare the given area with text Figure 2.5. We first integrate with respect to y from the parabola to the line:

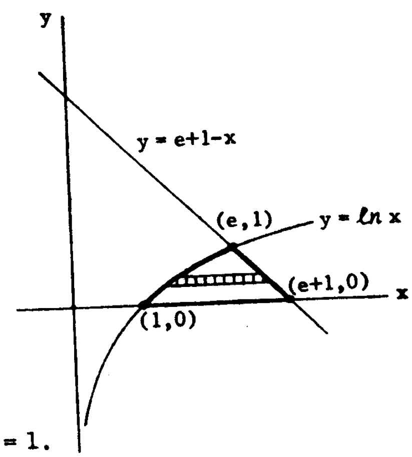
$$\int_{y=x^2}^{2x+8} dy = 2x + 8 - x^2$$

Then

$$\iint_A x\,dy\,dx = \int_{-2}^{4} x(2x + 8 - x^2)\,dx = \frac{2x^3}{3} + \frac{8x^2}{2} - \frac{x^4}{4}\Big|_{-2}^{4} = 36.$$

5-2.15

15. First verify the points of intersection as shown on the figure. Then compare the given area with text Figure 2.6. We integrate with respect to x from the $\ln$ curve to the straight line. If $y = \ln x$, then $x = e^y$, and on the line $x = e + 1 - y$, so the x limits are from e^y to $e + 1 - y$. Then we sum the horizontal strips from $y = 0$ to $y = 1$.



$$\int_{y=0}^{1}\int_{x=e^y}^{e+1-y} dx\,dy = \int_0^1 (e + 1 - y)\,dy = e + 1 - \frac{1}{2} - (e - 1) = \frac{3}{2}.$$