Boas 3-4.13

Given $\mathbf{A} = 4i - 3k$ & $\mathbf{B} = -2i + 2j - k$, find scalar projection of $\mathbf{A}$ onto $\mathbf{B}$

$$|\mathbf{A}| = A = (16+9)^{1/2} = \sqrt{25} = 5$$
$$|\mathbf{B}| = B = (4+4+1)^{1/2} = \sqrt{9} = 3$$

a) $\mathbf{A}\cdot\mathbf{B} = (-4.2) + 0 + 3\cdot 1 = -8 + 0 + 3 = -5$

$$A_B = \frac{\mathbf{A}\cdot\mathbf{B}}{B} = \frac{-5}{3} = \frac{-5}{3}$$

b) Projection of $\mathbf{B}$ onto $\mathbf{A}$

$$B_A = \frac{\mathbf{A}\cdot\mathbf{B}}{A} = \frac{-5}{5} = -1$$

c) Angle between $\mathbf{A}$ & $\mathbf{B}$:

$$\mathbf{A}\cdot\mathbf{B} = A\cdot B\cdot\cos\theta$$

$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{A\cdot B} = \frac{-5}{5\cdot 3} = -\frac{1}{3}$$

$$\therefore \theta = \cos^{-1}\theta = 109^\circ$$

Supplementry: Find $\mathbf{A}_B$ vector. First find unit vector along $\mathbf{B}$, $\mathbf{u}_B$. Then $\mathbf{A}_B = A_B\mathbf{u}_B$

$$\mathbf{u}_B = \frac{\mathbf{B}}{B} = \frac{-2i + 2j - k}{3} = -\frac{2}{3}i + \frac{2}{3}j - \frac{1}{3}k$$

Then $\mathbf{A}_B = \left(-\frac{5}{3}\right)\left(-\frac{2}{3}i + \frac{2}{3}j - \frac{1}{3}k\right)$

$$\mathbf{A}_B = \frac{10}{9}i - \frac{10}{9}j + \frac{5}{3}k$$

This is a vector in the direction of $\mathbf{B}$ with magnitude $-5/3$.