3-6.7

(a) First we multiply

$$\begin{pmatrix} -1 & 4 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} (-1)(1) + 4\cdot 2 \\ 2(-1) + (-1)(2) \end{pmatrix} = \begin{pmatrix} 9 \\ -4 \end{pmatrix} .$$

Then

$$(2 \quad 3)\begin{pmatrix} -1 & 4 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = (2 \quad 3)\begin{pmatrix} 9 \\ -4 \end{pmatrix} = (6).$$

Note that the result is a matrix of one row and one column, that is, one element. Verify that you get the same result by multiplying the first two matrices together and then the result times the third matrix.

3-6.8

We can write the equation as $r^T Ar = 30$ where $r = \begin{pmatrix} x \\ y \end{pmatrix}$, $r^T = (x \quad y)$ is the transpose of the matrix r, and $A = \begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix}$. Then

$$Ar = \begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (5x - 7y) \\ (7x + 3y) \end{pmatrix} .$$

This is a matrix of one column; note carefully that $(5x - 7y)$ is <u>one</u> element [compare part (a)]. Then

$$r^T Ar = (x \quad y)\begin{pmatrix} 5x - 7y \\ 7x + 3y \end{pmatrix} = \big(x(5x - 7y) + y(7x + 3y)\big) = (5x^2 + 3y^2).$$

This is a matrix of one row and one column, that is, one element. The equation says this one element equals 30. Thus we have

$$5x^2 + 3y^2 = 30 \qquad \text{or} \qquad \frac{x^2}{6} + \frac{y^2}{10} = 1,$$

which is the equation of an ellipse with semi-axes $\sqrt{6}$ and $\sqrt{10}$.