CHAPTER 4

PRECESSION OF THE EARTH'S AXIS

4-A. Introduction

The precession of the Earth's axis of rotation may be visualized as a slow, wobbling motion of the axis in space, pivoting at the Earth's center. What causes this?

Because the Earth rotates relatively rapidly around an axis, the shape of the Earth is not a perfect sphere, but rather, it is an oblate spheroid. This means the Earth is bulged at the equator. This is true in general for any object that rotates, and the faster the rotation, the greater the oblateness. Now, the Moon and the Sun exert gravitational forces on this bulge in such a way that the net result is to try and align the bulge with the plane of ecliptic, that is $\mathbf{F_g}$ net is perpendicular to the ecliptic. However, the Earth has rotational inertia, that is, angular momentum, **L**, which is a vector along the axis. Now the net gravitational force produces a torque that is always in a direction perpendicular to the axis of rotation and therefore perpendicular to **L**.

$$\mathbf{T = r \times F}$$

This torque adds a new component of angular momentum, $\Delta\mathbf{L}$, in the direction that it acts, that is perpendicular to **L**. When the vectors **L** and $\Delta\mathbf{L}$ are added together, the result is that the axis precesses. The gravitational torque, $\Delta\mathbf{L}$, cannot change **L** but only cause it to rotate in the direction of $\Delta\mathbf{L}$. That is, $\Delta\mathbf{L}$ cannot change the rate of rotation of the Earth. As a result of the precession, the Earth's rotational axis describes, or slides along, the surface of a right circular cone. See the diagram below.

The fact that the Earth's axis precesses has been known since ancient times. After Newton developed the laws of motion and gravity, he tried to account for the precession. He concluded that this could only happen if the Earth were oblate. However, at that time this had not yet been measured. Several years after Newton's death, it was discovered that the polar diameter of Earth was about 45 km shorter than the equatorial diameter, thereby verifying Newton's prediction.

4-B. Precessional Cycle

Currently, the Earth's axis precesses 360 degrees in **25,772** years and this length of time is called the precessional cycle or period. However, this motion does not change the 23.5^{o} tilt of the Earth's axis, that is, the obliquity of the ecliptic. Actually, the precession is more complicated than what we have described here, since the Moon is moving in orbit around the Earth, thereby changing its position in space relative to the Sun. Hence the combined gravitational pull of the Moon and Sun keeps changing, thereby introducing other harmonics into the precession. For example, the Moon's orbital motion around the Earth-Moon barycenter introduces a harmonic referred to as "nutation." In addition, the orbit if the Moon is tilted 5 degrees from the plane of the Earth's orbit. Planetary perturbations and the recession of the Moon caused by tidal effects are slowly lengthening the precessional cycle as well as altering the obliquity of the ecliptic.

Near the end of the 19th century, Simon Newcomb, a renowned Canadian-American astronomer and mathematician, calculated a value of 5,025.64 arcseconds per tropical century as the value for the general precession (*p*) in longitude (along the ecliptic). This was the generally accepted value until the advent of satellites, which provided more accurate observations, and electronic computers allowed more elaborate models to be calculated. Modern techniques such as VLBI (very long based interferometry) and LLR (lunar laser ranging) allowed further refinements, and the International Astronomical Union adopted a new constant value for p in 2000, and new computation methods and polynomial expressions in 2003 and 2006. The expression adopted for the accumulated precession is (Capitaine *et al*, 2003):

$$p_A = 5{,}028.796195 \times T + 1.1054348 \times T^2 + \text{higher order terms,}$$

in arcseconds, with T, the time in Julian centuries (that is, 36,525 days) since the epoch of 2000.

The rate of precession is the derivative of the above expression, namely:

$$p = 5{,}028.796195 + 2.2108696 \times T + \text{higher order terms.}$$

The constant term of this speed corresponds to one full precession circle in 25,771.58 years.

The precession rate is not a constant, but is (at the moment) slowly increasing over time, as indicated by the linear (and higher order) terms in T. In any case it must be stressed that this eqauation is only valid over a *limited time period*. It is clear that, if T gets large enough (far in the future or far in the past), the T^2 term will dominate and p will go to very large values. In reality, more elaborate calculations on the numerical model of the Solar System show that the precessional *constants* have a period of about 41,000 years, the same as the obliquity of the ecliptic. Note that the *constants* mentioned here are the linear and all higher terms of the formula above, not the precession itself. That is,

$$p = A + BT + CT^2 + \ldots$$

is an approximation of

$$p = a + b \sin (2\pi T/P),$$

where P is the 410-century period.

Theoretical models may calculate the proper constants (coefficients) corresponding to the higher powers of T, but since it is impossible for a (finite) polynomial to match a periodic function over all numbers, the error in all such approximations will grow without bound as T increases. In that respect, the International Astronomical Union chose the best-developed available theory. For up to a few centuries in the past and the future, all formulas do not diverge very much. For up to a few thousand years in the past and the future, most agree to some accuracy. For eras farther out, discrepancies become too large — the exact rate and period of precession may not be computed using these polynomials even for a single whole precession period.

The precession of Earth's axis is a very slow effect, but at the level of accuracy at which astronomers work, it does need to be taken into account on a daily basis. Note that although the precession and the tilt of Earth's axis (the obliquity of the ecliptic) are calculated from the same theory and thus, are related to each other, the two movements act independently of each other, moving in mutually perpendicular directions.

Precession exhibits a secular decrease as a result of tidal dissipation from 59"/a to 45"/a . Here a is in Julian years. This expression is valid during the 500 million year period centered on the present. After short-term fluctuations (tens of thousands of years) are averaged out, the long-term trend can be approximated by the following polynomials for negative and positive time from the present in "/a, where T is in billions of Julian years (Ga):

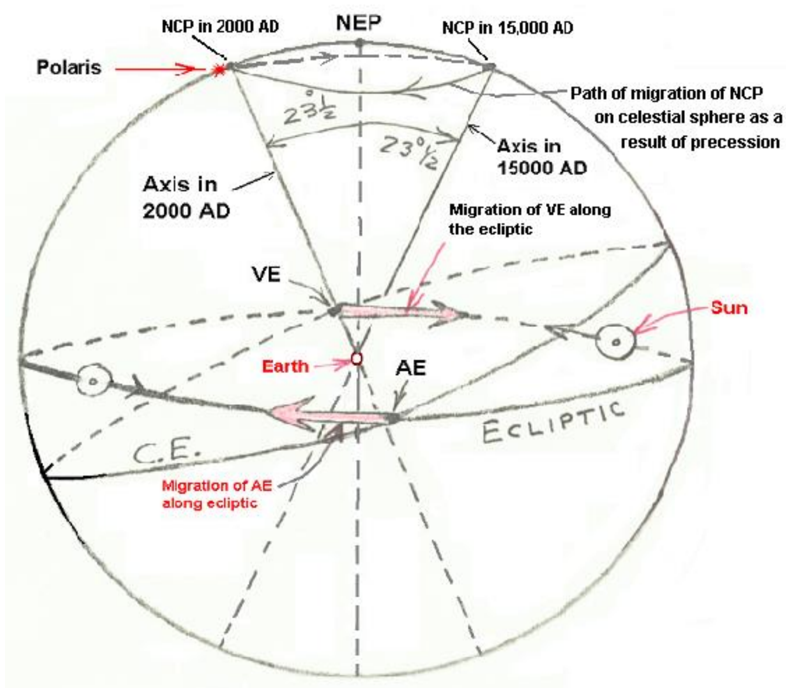
$$p^- = 50.475838 - 26.368583T + 21.890862T^2$$
$$p^+ = 50.475838 - 27.000654T + 15.603265T^2$$

Precession will be greater than p^+ by the small amount of +0.135052"/a between +30 Ma and +130 Ma. The jump to this excess over p^+ will occur in only 20 Ma beginning now because the secular decrease in precession is beginning to cross a resonance in Earth's orbit caused by the other planets.

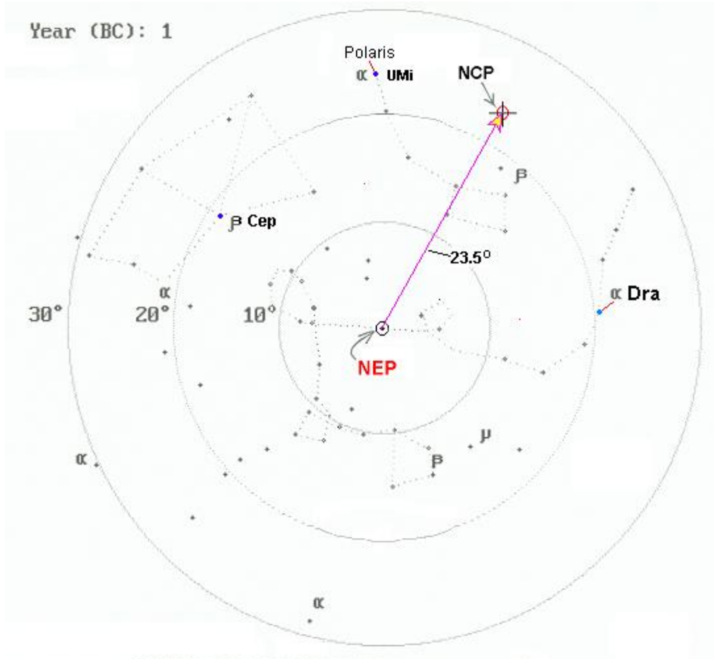
According to Ward (1982), when, in about 1,500 million years, the distance of the Moon, which is continuously increasing from tidal effects, has increased from the current 60.3 to approximately 66.5 Earth radii, resonances from planetary effects will push precession to 49,000 years at first, and then, when the Moon reaches 68 Earth radii in about 2,000 million years, the precessional cycle will be 69,000 years. This will be associated with wild swings in the obliquity of the ecliptic as well. Ward, however, used the abnormally large modern value for tidal dissipation. Using the 620-million year average provided by tidal rhythmites (rhythmic succession of sedimentation beds) of about half the modern value, these resonances will not be reached until about 3,000 and 4,000 million years, respectively. Long before that time (about 2,100 million years from now), due to the gradually increasing luminosity of the Sun, the oceans of the Earth will have vaporized, which will reduce tidal effects significantly. Furthermore, 4.5 billion years from now, the Sun will evolve into a red giant and likely destroy both the Earth and Moon.

4-C. Effects of precession

As a result of precession, the two points on the celestial sphere to which the Earth's axis points are changing relative to the fixed stars. By definition, these two moving points are the celestial poles. Hence, the NCP migrates in a circle of radius 23.5^o around the north ecliptic pole once every 26,000 years. See the diagram below. This means that the present North Star, which is called Polaris, has not always been the North Star.



Since the Earth's axis is always perpendicular to the plane of the celestial equator, the celestial equator moves with respect to the fixed stars also. This in turn means the equinoxes slide along the ecliptic over the approximate 26,000 year cycle as shown in the diagram. The chart below is a polar projection star-chart centered on the North Ecliptic Pole (NEP). On the chart, circles concentric about the NEP are drawn every 10 degrees The position of the North Celestial Pole (NCP) is indicated by the small red circle with a cross through it at the end of the red vector. The location of the NCP corresponds to the date given. As the NCP migrates in a circular path around the NEP, it is always a distance equal to 23.5^o from the NEP. The star chart shows the bright stars located along the path of the NCP around the NEP. At the present time, the NCP is located about 40 arcminutes from the star we call Polaris (α UMi), which is located about 24° from the NEP near the top of the chart. The NCP will be closest to Polaris in about the year 2140.



PRECESSION OF NORTH CELESTIAL POLE

Another effect of the precession enters into computing the length of the year. The determination of the latter depends on what reference is used to judge the revolution of the Earth in its orbit. If one uses the fixed stars as reference points, one defines what is called the sidereal year. Observations yield the length of the sidereal year to be 365.2663 days or 365d 6h 9m 10s. The sidereal year is the time for the Earth to revolve exactly 360 degrees in orbit around the Sun and is the true period of revolution of the Earth in orbit.

If we used this for the length of the calendar year, the result would be that Sun would arrive at the vernal equinox earlier every year, since the vernal equinox migrates westward as a result of precession. This means the first day of spring would come earlier and earlier in the calendar every year. One does not want this. Therefore, we need a calendar that keeps the first day of spring occurring on the same day every year.

To accomplish the latter, something called the tropical year or year of the seasons is defined as the length of time it takes the Earth to revolve in orbit so that the Sun appears to move around the ecliptic from the vernal equinox back to the vernal equinox again, even though the latter has moved. This takes 365.2422 days or 365d 5h 48m 46s. This is because the vernal equinox, as a result of precession, is moving 50.26 arc-seconds westward every year relative to the stars. So, the Sun arrives back at the vernal equinox before it arrives back at a fixed star.

The Earth tales about 20 minutes to revolve through an angle of 50.26 arc-seconds in its orbit, so the year of the seasons, or Tropical year, is 20 minutes shorter than the sidereal year.

References

Capitaine, N., Wallace, P. T. and Chapront, J. 2003. *Astronomy & Astrophysics* **412**, 567–586.

Ward, W. R. 1982, "Comments on the long-term stability of the Earth's obliquity", *Icarus* **50**, 444.