

# CHAPTER 11

## Stellar Photometry

### 11-1. Brightness in General:

Brightness may be defined as the amount of radiant energy received from a light source per second within a certain bandpass. By radiant energy is meant the energy carried by electromagnetic radiation or waves. A bandpass is usually an interval of contiguous wavelengths of the total electromagnetic spectrum. Brightness also depends on the size of the telescope, the area of the detector or sensor, and the sensitivity of the detecting device that is used.

The brightness at a unique wavelength is called monochromatic brightness,  $b_\lambda$ . The brightness in a bandpass is known as polychromatic brightness,  $b_{\Delta\lambda}$ .

### 11-2. Apparent brightness:

Apparent brightness is the brightness of an object as seen from the Earth. That is, how bright an object appears to be depends on the distance of the object from the observer. One cannot use apparent brightness to compare stars as to which are truly bright or faint, since stars are at different distances from the Earth.

Hipparchus, circa 150 BC, devised what is called the magnitude system for expressing stellar brightness. He divided all the stars visible to the unaided eye into 6 different classes of brightness. He identified a number of stars that he considered were the brightest that could be seen and called them 1st magnitude (designated as  $m=1$ ). The faintest stars visible to unaided eye he classified to be 6th magnitude ( $m=6$ ). The remaining stars were assigned magnitudes from 2 to 5.

Note: These are apparent magnitudes because they are an attempt to measure brightness as seen from Earth. Furthermore, they are apparent **visual** magnitudes, since the human eye only detects or is sensitive to a limited portion of all the radiations emitted by an object. This portion is called the **visible spectrum or the visible bandpass**.

After the invention of the telescope, fainter stars could be seen and these have been assigned magnitudes  $>6$ . With today's technology, stars as faint as  $m=28$  can be detected with very sensitive electronic devices. Furthermore, the magnitude system has been defined more precisely so that fractions of a magnitude may be assigned, and more objectively, using instruments rather than the human eye.

In the modern magnitude system, a **step or difference of 5 magnitudes ( $\Delta m=5$ ) is defined to represent a brightness ratio of exactly 100**. That is, we receive 100 times more light energy per second from a first magnitude star than we do from a 6<sup>th</sup> magnitude star, and we receive 100 times more light energy from a 4<sup>th</sup> magnitude star than we do from a 9<sup>th</sup> magnitude star.

A difference of 1 magnitude ( $\Delta m=1$ ) corresponds to a brightness ratio equal to the fifth root of 100, which is approximately 2.512. The brightness ratio of two stars that differ in magnitude by any amount is then 2.512 raised to a power equal to their difference in magnitude,  $\Delta m$ , that is,

$$B_1/B_2 = (2.512)^{\Delta m}.$$

If we now assume that the faintest stars seen by the unaided eye are exactly 6th magnitude, then some of the stars that Hipparchus had called 1st magnitude were actually brighter than 6th by more than 100 times. This necessitated introducing negative magnitudes, so that the apparent magnitude of the brightest star, Sirius, is now  $m = -1.47$ .

The magnitude system may also be assigned to any object, including the Sun ( $m = -27$ ), Moon ( $m = -12.5$ , when full), planets (Venus gets as bright as  $-4.4$ ), comets, galaxies, etc. If the Sun were viewed from the outskirts of the Solar System it would appear to have an apparent magnitude of about  $-2$ .

### 11-2. Absolute, Intrinsic, or true Brightness

This is the true brightness of an object, independent of its distance. The intrinsic brightness of a star depends only on its:

1. Surface brightness,  $B_s$ .
2. Radius,  $R$ , or surface area.

### 11-3. Surface Brightness ( $B_s$ )

This is the total amount of radiant energy emitted from, passing through, or falling on a square centimeter per second.

The surface brightness or **flux** of a star,  $B_s$ , expresses how much energy is radiated into space from every square centimeter of a star's surface per second. The surface brightness of a star depends only on the surface temperature of the star,  $T$ , and is given by the **Stefan-Boltzmann Law**:

$$B_s = \sigma T^4$$

where  $\sigma$  (or lower case Greek sigma) is a constant of proportionality, which must be measured in the laboratory. That is, the flux of a star is directly proportional to the fourth power of the absolute temperature and no other physical property of the star. The factor  $\sigma T^4$  is also known as the integrated or bolometric surface brightness, or bolometric flux of a star.

### 11-4. Luminosity

This is an expression of the total amount of radiant energy that a star emits into space every second. The symbol for luminosity is  $L$ . Luminosity is a way of expressing the intrinsic or true brightness of a star. Therefore, luminosity depends only on the surface temperature and radius of the star and it does not depend on the distance of the star. That is

$$L_s = B_s \times (\text{surface area}),$$

where the surface area depends on the radius of the star, such that  $\text{Area} = 4\pi R_s^2$ . Then:

$$L_* = 4\pi R^2 \sigma T^4 \quad (11-4.1)$$

Here  $R$  is the radius of the star and  $T$  is the surface temperature. The luminosities of stars are usually given in terms of the Sun's luminosity, e.g.  $L_* = 8.00L_{\odot}$ .

A commonly used unit of luminosity is the watt. For example, the Sun's Luminosity is

$$L_{\odot} = 3.90 \times 10^{26} \text{ watts.}$$

Luminosities for other stars are usually given in terms of the Sun's luminosity,  $L_{\odot}$ . That is, the total amount of energy that the Sun emits per second is called 1 solar unit of luminosity. A star that has a luminosity 100 times greater than the Sun's would be written as  $L_* = 100L_{\odot}$

As the total light from a body travels outwards into space, it must pass through successive, concentric spheres of larger and larger surface area. Hence, the brightness of the light must decrease with distance from the source. Since the area of a sphere depends on the square of its radius, the brightness must be inversely proportional to the square of the distance (which is the radius of a sphere) from the light source. In other words, the brightness of light obeys an inverse square law, just like gravity does

Hence, very distant stars are going to appear faint or have large magnitudes while nearby stars are going to appear to be very bright or have small magnitudes. So apparent magnitudes cannot indicate which stars are intrinsically bright and which are intrinsically faint. To determine this, we must eliminate the distance factor when assigning magnitudes. The inverse-square law makes it possible to calculate what magnitude would be seen at any distance, if we measure the magnitude for a known distance.

## 11- 5. Absolute magnitude, $M$ .

**Absolute magnitude** is also an expression of intrinsic brightness. **It is the magnitude of an object when seen from a distance of 10 parsecs.** However, the absolute magnitude scale is a relative scale of absolute or intrinsic brightness. Astronomers use absolute magnitudes to express which stars are truly bright and which are truly faint, because distance is no longer a variable.

The absolute magnitude scale works the same way the apparent magnitude scale works. For example, a star that is 100 times more luminous than the Sun would have an absolute magnitude that is 5 magnitudes brighter than the Sun's absolute magnitude (remember, a step of 5 mags. is defined to correspond to a brightness ratio of exactly 100). Since the Sun's absolute magnitude is approximately +5 (4.79 to be exact), a star with a luminosity 100 times the Sun's would have a value of  $M_* = 0$ .

Absolute magnitude,  $M$ , is a number that can only be computed, not measured. To compute  $M$  for a star we must first:

1. Measure the apparent magnitude of the star.
2. Determine the distance of the star by measuring its parallax..
3. Use the inverse-square law to compute the magnitude the star would have if seen from 10 parsecs.

Now, the distance of a star may be calculated using trigonometry, if a very small angle called the parallax of the star can be measured. Knowing the distance and apparent magnitude of a star, one can use the inverse square law to compute what its magnitude would be at 10 pc using the inverse square law for the diminution of brightness. Let  $B_M$  be the brightness of an object when observed from a distance of 10pc and  $b_m$  the brightness of the same object when observed from a distance  $d$ . Then the inverse square law gives:

$$\frac{b_m}{B_M} = \left( \frac{10}{d} \right)^2 \quad (11-5.1)$$

But according to the definition of the magnitude scale,

$$b_m / b_M = 2.512^{(M-m)} = [10^{0.4}]^{(M-m)}$$

So,

$$(10/d)^2 = [10^{0.4}]^{(M-m)}$$

Take the log of both sides:

$$2(\log 10 - \log d) = (M-m) (0.4) \log 10$$

$$(2-2\log d)/0.4 = M-m$$

$$5-5\log d = M - m$$

Solve for M:

$$\mathbf{M = m + 5 - 5\log (d)} \quad (11-5.2)$$

Here  $d$  must be expressed in parsecs. Or, since  $d=1/\pi$ ,

$$\mathbf{M = m + 5 + 5\log (\pi)}, \quad (11-5.3)$$

where  $\pi$  is the parallax in arcseconds. However, this can be done only for about 2,000 stars, all of which are within 100 pc of the Sun. It has been found that the absolute magnitudes of stars range from -10 (the intrinsically brightest stars) down to +18. The Sun's absolute visual magnitude is +4.79, making it an average star when compared with the other stars.

Example: The star Rigel has an apparent magnitude that has been measured to be 0.18 and it is known that the star also has an absolute magnitude that is -6.60. What is the distance of the star in parsecs.

First calculate the distance modulus of the star,  $m-M = 0.18-(-6.60) = 6.78$ . When the distance modulus is greater than 0.00, the star has a distance greater than 10 parsecs. If the distance modulus is negative, that is, less than 0.00, the star is closer than 10 parsecs. If the distance modulus is exactly 0.00, then  $m=M$ , and the star has a distance of 10 parsecs.

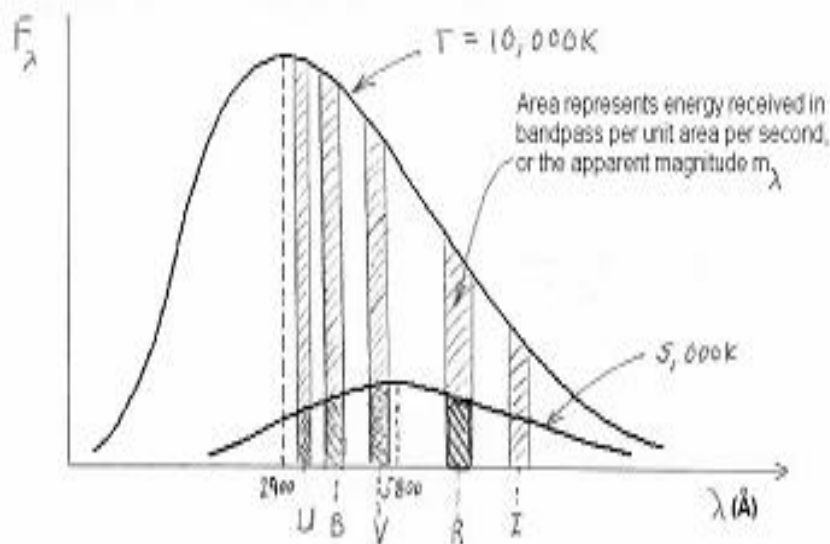
Now  $[(m-M)/5] + 1.00 = [(6.78)/5] + 1.00 = 1.37 + 1.00 = 2.37$ .

Hence the distance of Rigel is  $10^{2.37} = 234$  parsecs.

### 11-6. Color Magnitudes

The flux emanating from the surface of an incandescent body, such as a star, has a wavelength dependence given by Planck's Law. This is shown in the diagram below for a surface temperature of 10,000 K. Therefore, one needs to specify the wavelength interval or bandpass over which the measurement of a magnitude has been made. The human eye detects what is called "visible" light or electromagnetic radiation from a wavelength of about 400 nanometers to 700 nm (4000 to 7000 Å). Hence, magnitudes measured by the eye are called "visual" magnitudes." One may construct a photometer to measure the brightness over any wavelength interval or bandpass desired.

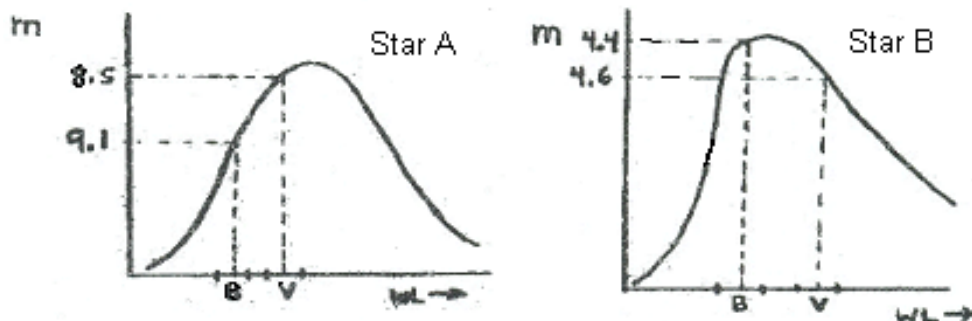
Magnitudes that are measured for a certain defined bandpass are called color magnitudes. One of the standard color magnitude systems used by astronomers is the Johnson and Morgan **U, B, V, R, I** system. See the schematic below. The bandwidths for each of these color magnitudes is several decades of nanometers in the ultraviolet, blue, yellow-green, red, and near infrared portions of the EM spectrum. Note that for 10,000 K, a star is brighter in blue (smaller magnitude) than it is in the visible or red bandpasses.



Measured values of the color magnitudes are also designated as  $m$  with a subscript identifying the bandpass, e. g.,  $m_v$  or  $m_B$ .

### 11-7. Color Index:

A **color index** is the difference of two color magnitudes, e.g., B-V, U-B, etc. A color index



that is used very often is the B-V index. The diagram below illustrates the spectral distributions for two stars with different temperatures. The B-V color index for star A is  $9.1 - 8.5 = +0.6$ , whereas the B-V color index for star B is  $4.4 - 4.6 = -0.2$ . Now star B is the hotter star with a surface temperature of 20,000 K and star A is cooler with a surface temperature of 5500 K. Notice that the hotter star has a negative color index while the cooler star has a positive color index. That is, hot stars tend to be brighter in the blue part of the spectrum than in the visual or red part of the spectrum. For such stars,  $B < V$  numerically. Cooler stars are brighter in the visual bandpass than in the blue bandpass so  $V < B$  numerically. Hence, color indices convey useful information about a star's spectrum and temperature. Values of B-V have been calibrated to directly indicate the temperature of a star. An abbreviated table is given below.

| B-V   | Temp (K) |
|-------|----------|
| -0.30 | 33,000   |
| -0.15 | 14,800   |
| 0.00  | 9,600    |
| +0.50 | 6,500    |
| +1.00 | 4,800    |
| +1.52 | 3,600    |

### 11-8. Bolometric Magnitudes

A bolometric magnitude,  $m_{\text{bol}}$  (or  $M_{\text{bol}}$ ), is the magnitude found by measuring the observed flux over the entire EM spectrum. Needless to say, this is very difficult to do. Bolometric magnitudes are always brighter than magnitudes measured for some bandpass (smaller in value). For example  $M_V$  for the Sun is +4.79, whereas its absolute bolometric magnitude is  $M_{\text{bol}} = +4.72$ .

As the radiation from a body travels outwards from the surface, it must pass through successive concentric spheres of larger and larger surface area. Using the conservation of energy for the luminosity of a star we have

$$L_* = 4\pi R^2 \sigma T^4 = 4\pi r^2 \int F_\lambda(r) d\lambda = 4\pi r^2 F_{\text{bol}}(r), \quad (11-8.1)$$

where  $F_{\text{bol}}(r)$  is the total integrated (over all wavelengths) or bolometric flux arriving on a sphere at a distance  $r$  from the star. Therefore,

$$F_{\text{bol}}(r) = L_*/4\pi r^2. \quad (11-8.2)$$

Both absolute bolometric magnitude and luminosity are measures or expressions of intrinsic brightness. Absolute magnitude does so on a relative scale (dimensionless), while luminosity is on an absolute scale (watts). The relationship between the two is

$$L_{*A} / L_{*B} = 2.512^{(M_{bolB} - M_{bolA})} \quad (11-8.3)$$

or ,

$$(M_{bolB} - M_{bolA}) = 2.5 \log(L_{*A} / L_{*B}) \quad (11-8.4)$$

## 11-9. Bolometric Corrections

The bolometric correction is defined as

$$BC = M_{bol} - M_V \quad (11-9.1)$$

For example, the bolometric correction for the Sun is  $4.72 - 4.79 = -0.07$ . Bolometric corrections are always negative and are usually computed from theory. They may be found tabulated in various sources, even on line.

## 11-10. Distance Modulus:

Distance modulus is an indicator of the distance of an object expressed in terms of its absolute and apparent magnitudes. More specifically, distance modulus is defined as **m-M**. If the distance modulus is 0, the object is 10 parsecs distant. If m-M is less than zero or negative, this means the object is closer than 10 parsecs. If m-M is positive, then the object is farther than 10 parsecs. Distance modulus is actually a logarithmic index of distance.

## 11-11. Selective Absorption and Reddening

In addition to distance affecting the apparent brightness of an object, further dimming is caused by absorption within the interstellar medium. In general, the more distant the object, the greater the amount of absorption, but it also depends on the line of sight to the object through the galaxy. The absorption is also wavelength dependent, with the result that an object is reddened, just as the Sun and Moon are when seen towards the horizon. The result is that the measured color index,  $(B-V)_m$ , is increased relative to its intrinsic value,  $(B-V)_o$  or  $(B-V)_i$ . Intrinsic values may be computed from the black body theory of radiation and may be found tabulated for a given temperature. Let  $\Delta m_V$  be the diminution of the star in magnitudes and  $E_{B,V}$ , **the color excess**, defined as

$$E_{B,V} = (B-V)_m - (B-V)_o \quad (11-11.1)$$

A general rule that is sometimes used is

$$\Delta m_V = 3E_{B,V} \quad (11-11.2)$$

Hence, when computing the distance of a star, one must take into account the interstellar absorption. This requires knowing the spectral type or temperature of the star in order to look up the intrinsic value of the color index,  $(B-V)_0$ , and the determination of B and V in order to find  $(B - V)_m$ . The value of  $\Delta m_v$  is always a positive number that must be subtracted from the observed value of V or  $m_v$