CHAPTER 11
The General Theory of Relativity and Black Holes

Early in the 19th century, the German mathematician, Riemann, developed the mathematics that describes the properties of curved spaces of any dimension, including what are known as hyperspaces (spaces of more than 3 dimensions).

The terms "manifold" and "continuum" are synonyms for space.

By 1915, A. Einstein developed the General Theory of Relativity. In this theory, Einstein developed a new way of representing gravity that is superior to Newton's concept. It is superior since it is able to account for more phenomena, including many phenomena encountered in cosmology and wherever there are very dense objects, such as black holes.

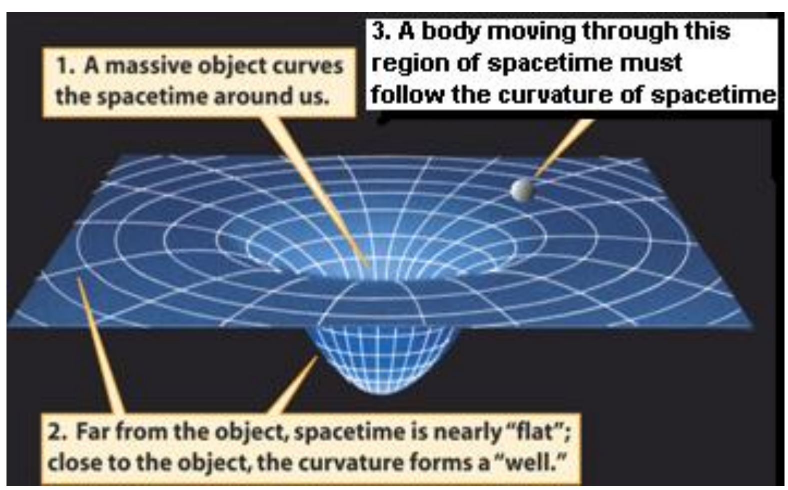
In order to explain certain physical phenomena that were challenging theoreticians at that time, Einstein introduced the idea of the Space-Time Continuum. That is, space and time make up a 4 dimensional hyperspace (manifold) or geometric fabric of the universe. Einstein applied the equations of Riemannian geometry to describe the properties of the S-T continuum.

In general relativity, one imagines that empty space is at least 4D and has geometric properties. One of these properties is that the S-T continuum is elastic and can stretch or be deformed. This means the properties of empty space may be altered. In his special theory of relativity published in 1905, Einstein had already introduced the idea that space and time are **not absolute things but depend on your motion relative to the thing you are measuring.**

In the general theory of relativity, Einstein also introduced the idea that matter exists in the universe in such a way that it is embedded in the geometric fabric or S-T continuum and affects the shape of the S-T continuum. More specifically, matter bends the S-T continuum or gives it curvature. As something travels from one point in space to another, it must follow the curvature of the S-T continuum, even light. In an analogous way, as we travel from one place on the Earth to another, we must follow the curvature of the Earth.

Though we can visual the curvature of the Earth's surface, which is 2 dimensional, we can not see or visualize the curvature of our 4D universe. However, the geometry of such a universe can be described by the mathematics developed by Riemann.

Now, since movement along a curved path is the result of a force, movement along the curvature of the S-T continuum is equivalent to having a force act on you. Hence, Einstein interpreted the curvature of the S-T continuum to be gravity.



According to the general theory of relativity, gravity is the curvature of the space-time continuum, and mass causes this curvature.

This is a different way of representing gravity and replaces Newton's concept of gravity being a force of attraction acting over the space between objects.

Einstein's theory is also able to explain phenomena that can not be explained by Newton's concepts. For example, Einstein's theory predicts that light is affected by gravity, but Newton's Law does not. The prediction that light is affected by gravity has been verified astronomically, thereby giving support to this theory. During a total eclipse of the Sun, it was discovered that the stars in the sky that were near the Sun had their positions altered by exactly the amount that Einstein had predicted, since the light from these stars had to pass through the more highly curved region of space-time surrounding the Sun, before reaching an observer on the Earth. The general theory of relativity makes other predictions that have also been verified.

In essence, what the general theory of relativity does is equate the laws of physics to the geometry of the universe. In other words, the laws of physics describe the geometry of our universe.

Saying the above another way, when the laws of physics are expressed using the mathematics developed by Riemann, they are the same as the Riemannian equations that describe the geometry of a curved space-time continuum.

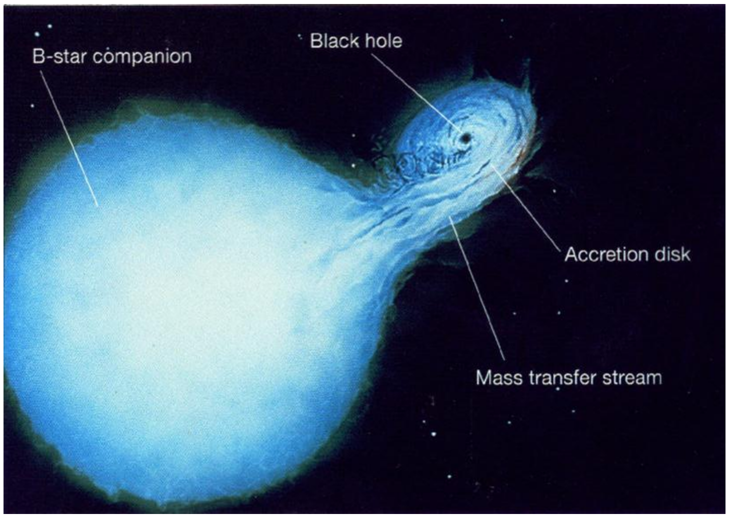
It is only by applying Einstein's theory of gravity to collapsing **stars of large mass that we are able to understand what happens to them.**

Such collapsed stars bend the space-time continuum around themselves so severely that this region of space becomes like a gravitational hole in the universe. The escape velocity from such a collapsed star is greater than the speed of light. This means light can not escape from the star so that it would be black. Thus, the name "black hole" came about.

It was the German mathematician Karl Schwarzschild who first found a solution to Einstein's equations that indicated these properties for black holes. The size of a star when it shrinks to the point where light can not escape is called the Schwarzschild or "critical" radius of a star. To find this solution, Schwarzschild assumed the star was not rotating, otherwise the problem was more difficult. But such an assumption leads to the result that the star must collapse to infinite density. Obviously, this is remained a problem.

In the 1963, the New Zealand theoretical physicist, Roy Kerr, was able to find a solution to Einstein's equations for a rotating black hole. In this case, the star does not have to collapse to zero radius. Instead, the star opens a connection or bridge between one place in the universe to another far away in time and or space. This bridge or connection is called a "wormhole" and has been used in many science fiction stories, such as "Deep Space Nine," “Star Trek,” and a Disney movie titled "The Black Hole." The reality of Kerr's solution is not fully understood nor is their any observational phenomena that may be associated with this interpretation of the solution. This is a frontier of investigation.

The question is, “do black holes really exist?” The answer appears to be yes. There are binary stars that have been found to consist of an ordinary blue giant and a very small but massive companion that seems to be a black hole. As the blue giant evolves and expands, some material is captured by the small, massive companion star and forms what is called an accretion disk around that star. See the diagram to the right. This material is slowly spiraling into the black hole whereupon this material is compressed and heated to several million Kelvins. At this temperature the material emits



copious amounts of x-rays and these x-rays are observed to go away when the black hole is eclipsed by its blue giant companion. An analysis of the light curve for such a system renders the size of the companion and an analysis of the radial velocities of the stars reveals the mass of the black hole. The size, mass, and presence of an x-ray emitting accretion disk all strongly suggest that it is a black hole.