Extensions of One-Dimensional Gray-level Nonlinear Image Processing Filters to Three-Dimensional Color Space

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ABSTRACT

This paper describes the development of computer simulation models to evaluate the filter algorithms in different color spaces. The objective of this project is to study the extension of filters from a one-dimensional gray-level space to a three-dimensional color space. The principal task of this project was to conduct and evaluate mathematical extensions of filters and determine which combination of filters and color spaces presents the best image quality. Results have shown that the use of the Rank Order filter with the YIQ color spaces provides the highest signal-to-noise-ratio. Therefore, the use of Rank Order filters in YIQ color spaces would be the most effective in the noise smoothing of images.

Keywords: color image smoothing, nonlinear imaging filters, color image processing

1. BACKGROUND

Color is that attribute of light-energy which is related to the wavelength. It is well known that color carries a very important part of information regarding objects of interest in an image [1]. The attention to the representation of color (color-models) and color-based segmentation of images has increased over the recent past few years. Applications include (i) wound-image processing, (ii) TB-screening, and (iii) Segmentation of Fascia.

Smoothing processing is one of the common methods in image enhancement that clears up the image noise, and its theories are all aimed at monochrome image. Using color space transformation of color image can not only gain more useful information, but also use the processing methods for a monochrome image. Using color space transformation based on L-H-S, the experiment for color image smoothing processing is made. The experimental results have shown that the method can availably cleans up the noise infection of color image and improve the imaging quality [2].

Other techniques have explored the theory of vector bundles over Riemannian manifolds in order to smooth multi-valued images [3]. And these frameworks have considered standard PDE's used in image processing as generalized heat equations, related to the geometries of the base manifold, given by its metric and the subsequent Levi-Cevita connection and of the vector bundle, given by a connection. As a consequence, the smoothing is made through a convolution with a 2D kernel, generalizing Gaussian, Beltrami and oriented kernel. In particular, extensions of the oriented kernel have been constructed, and illustrated with applications to color image smoothing.

Additionally, simplification and merging techniques based on an initial region partition of color images: a region adjacency graph (RAG) have been proposed; as is a general scheme that can be employed for both color image simplification and/or segmentation [4]. A new filtering algorithm of RAG was presented and included within a merging algorithm. The region models associated to each node are simplified and moreover the RAG is simplified by merging similar nodes.

In this work, the development of computer simulation models to evaluate the filter algorithms in different color spaces was undertaken. The objective of this project was to study the extension of filters from a one-dimensional gray-level space to a three-dimensional color space. The principal task was to conduct and evaluate mathematical extensions of filters and determine which combination of filters and color spaces presents the best image quality. Results have shown, and discussion will support, that the use of the Rank Order filter with the YIQ color spaces provides the highest signal-to-noise-ratio. Therefore, the use of Rank Order filters in YIQ color spaces would be the most effective in the smoothing of images.

2. INTRODUCTION

In this project, the development of computer simulation models to evaluate the filter algorithms in different color spaces was executed. The objective was to study the extension of filters from a one-dimensional gray-level space to a three-dimensional color space. The principal task was to conduct and evaluate mathematical extensions of filters and determine which combination of filters and color spaces yields the best image quality results.

The filters that were investigated are listed below:

- Weighted order statistics
- Stack
- Spatial-rank order selection
- Nonlinear mean
- Teager
- Polynomial
- Rational

The three-dimensional color spaces that were evaluated are listed below:

- RGB – Red Green Blue
- HSV (HSB) – Hue Saturation Brightness
- XYZ (CIE) – International Commission on Illumination (French)
- YIQ – Luma In-Phase Quadrature

The signal to noise ratio is used to compare and contrast the performance of each combination of filters with color spaces. Every possible combination of filters and color spaces were tried along with matrix multiplications (dot product and cross product). The performance of each combination was contrasted with the gray-level counterparts.

All the filters used in this investigation are nonlinear filters. A nonlinear filter is an operation where each filtered pixel y(m,n) is a nonlinear function of x(m,n) and its neighbors. In this investigation, a window of 3 x 3 is used to determine the neighbors of each pixel.

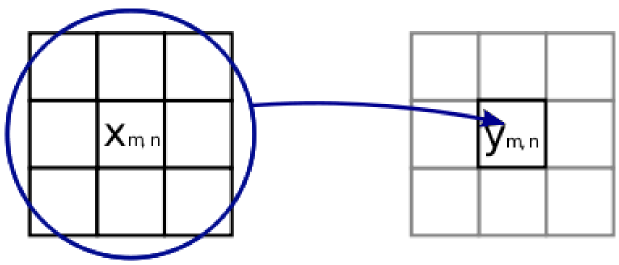

Figure 1: 3 x 3 Window (neighboring pixels)

The equation used for the output of the Weighted Order Statistic (WOS) filter is:

$$Y(m,n) = \text{median}\{ (w1 \blacklozenge x(m-1, n-1), (w2 \blacklozenge x(m, n-1), (w3 \blacklozenge x(m+1, n-1), (w4 \blacklozenge x(m-1, n), (w5 \blacklozenge x(m, n), (w6 \blacklozenge x(m+1, n), (w7 \blacklozenge x(m-1, n+1), (w8 \blacklozenge x(m, n+1), (w9 \blacklozenge x(m-1, n+1) \} \quad (1.0)$$

In terms of computing the output pixel value, the median of all the pixels in the window is selected where certain pixel values are weighted or repeated. The pixels at the four corners have a weight of 1. The center pixel has a weight of 3 while the others have a weight of 2. The median value is then selected as the output. The stack filter is the same as the WOS filter except that none of the pixel values are weighted. Therefore, it takes the median value of the 9 pixels.

The equation used for the output of the Nonlinear mean filter is:

$$Y(m,n) = \{x(m-1, n-1) + x(m, n-1) + x(m+1, n-1) + x(m-1, n) + x(m, n) + x(m+1, n) + x(m-1, n+1) + x(m, n+1) + x(m-1, n+1)\}/9 \quad (2.0)$$

The output is equal to the sum of its neighboring pixels divided by the total number of neighbors. Because the window is a 3x3 window, the number of neighbors is 9. Therefore, the output is the mean of the pixel values within the window.

The equation used for the Teager filter is:

$$Y(m,n) = \{3x^2(m,n) - \tfrac{1}{2}[x(m+1,n+1) * x(m-1,n-1)] - \tfrac{1}{2}[x(m+1,n-1) * x(m-1,n+1)] - x(m+1,n) * x(m-1,n) - x(m, n+1) * x(m, n-1)\} \quad (3.0)$$

In this filter, the center pixel is squared and multiplied by three. It is then subtracted by four values: (1) half of upper left corner pixel value multiplied by the lower right corner pixel value; (2) half of the upper right pixel value multiplied by the lower left pixel value; (3) the middle pixel value in the top row multiplied by the middle pixel value in the last row; and (4) the middle pixel value in the first column multiplied by the middle pixel value in the last column.

The equation used for the polynomial filter is:

$$y(m,n) = x(m,n) + \lambda x(m,n) \quad (4.0)$$

$$y(m,n) = [x(m-1,n) - x(m+1,n)]^2 * [2x(m,n) - x(m-1,n) - x(m+1,n)] + [x(m,n-1) - x(m,n+1)]^2 * [2x(m,n) - x(m,n-1) - x(m,n+1)] \quad (4.1)$$

$$\lambda = \text{intensity of enhancement} \quad (4.2)$$

This filter is a polynomial function which uses multiplication and powers to remove the noise in an image. This type of filter is more of an edge detection filter whereas; more emphasis is put on the edges in the image.

The equations used for the output of a rational filter are:

$$y(m,n) = x(m,n) + \lambda[vn1(m,n) * cn1(m,n) - vn2(m,n) - cn2(m,n)] \quad (5.0)$$

$$vn1(m,n) = 2x(m,n) - x(m,n-1) - x(m,n+1) \quad (5.1)$$

$$vn2(m,n) = 2x(m,n) - x(m-1,n) - x(m+1,n) \quad (5.2)$$

$$cn1(m,n) = gn1(m,n) / [k\ gn1^2(m,n) + h] \quad (5.3)$$

$$cn2(m,n) = gn2(m,n) / [k\ gn2^2(m,n) + h] \quad (5.4)$$

$$gn1(m,n) = [x(m,n+1) - x(m,n-1)]^2 \quad (5.5)$$

$$gn1(m,n) = [x(m+1,n) - x(m-1,n)]^2 \quad (5.6)$$

$$k = 1/(2\ g0) \quad (5.7)$$

$$h = g0 / 2 \quad (5.8)$$

This filter embodies a complex algorithm. There are several equations that are required in order to implement this filter. The value of g0, the position of resonance was set to 1.

These filters are applied to images of different color spaces such as YIQ, RGB, XYZ, and HSV. A color space is an abstract mathematical model which describes the way that the colors in the image can be represented as tuples of numbers, usually 3 color components. In this investigation, only the color spaces with 3 components are used. There are color spaces with four color components such as CMYK. RGB is an additive color model where red, green, and blue light are added together to reproduce a broad array of colors. RGB is the most commonly used color space in technology. Most computer monitors and televisions use RGB. YIQ is the color space used by the NTSC color TV system. The Y stands for luma, I for in-phase, and the Q stands for quadrature. There are very few televisions today that perform true YIQ decoding. The XYZ color space was created by the International Commission on Illumination (CIE). This color space is very similar to RGB in that its tri-stimulus values X,Y,Z are roughly red, green, and blue but are not observed as red, green, or blue. Instead, they are thought of as derived parameters from the red, green, and blue colors. The HSV color space stands for Hue – Saturation – Value. It is also known as HSB where the B stands for brightness. HSV is one of the most common cylindrical coordinate representations of points in an RGB color model where the geometry of RGB is rearranged in an attempt to make it more perceptually relevant than the Cartesian representation. This color model is common in computer vision applications.

As one can see, the use of different filters with different color spaces and types of mathematical computations (magnitude, cross product, dot product) can improve or decrease the quality of an image and remove some noise. In this project, the principal task is to find the combination that produces an image with the best quality. In order to do so, one must use the signal to noise ratio. The signal to noise ratio quantifies how much a signal has been corrupted by noise. It is defined as the ratio of signal power to the noise power corrupting the signal. In this project, the gray-level image that is filtered is compared to the filtered color to grey level image using the signal to noise ratio. The alternate definition of signal to noise ratio was used which is the ratio of mean to standard deviation of a signal. The equation for SNR in dB is:

$$SNR_{dB} = 20\log\frac{\mu}{\sigma} \quad (6.0)$$

Therefore, the numerator inside the log function is the mean of the signal, meaning the mean of the grey-level filtered pixel values. The denominator inside the log function is the standard deviation of the filtered color to gray image pixel values subtracted by the filtered gray-level image pixel values. The result will help determine the quality, which will help conclude which combination of filters and color spaces produces the best image in terms of quality and reduction in noise.

3. PROCEDURE

The model is shown in the diagram below:

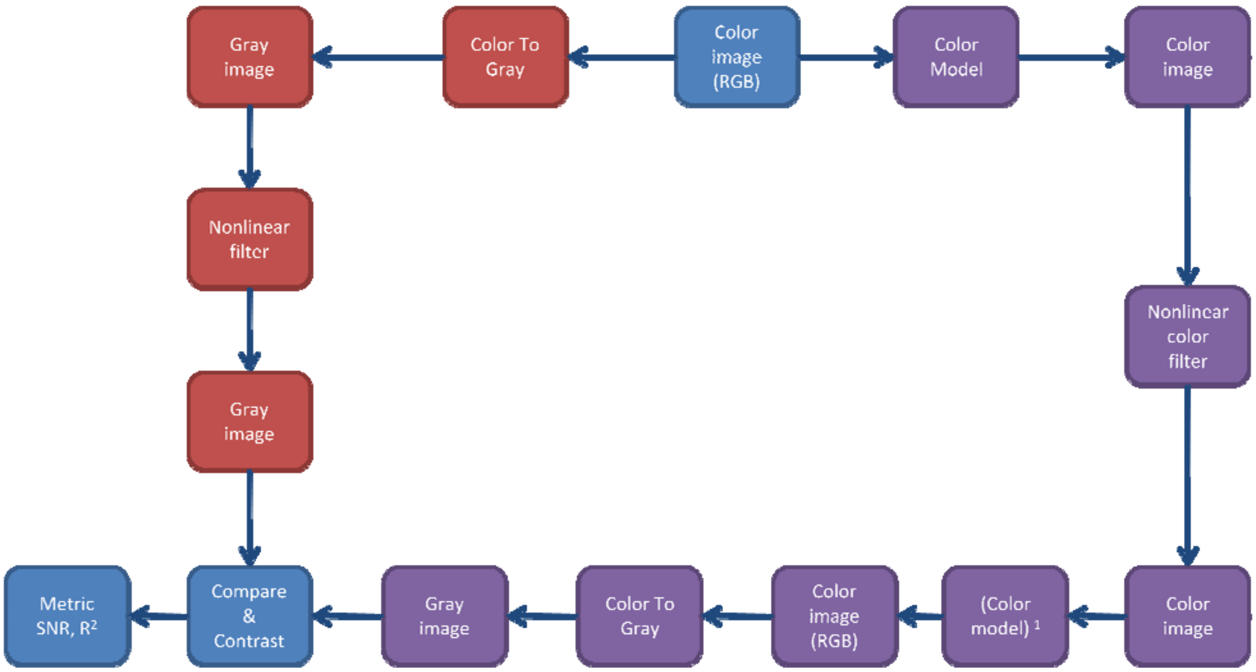

Figure 2: Software Architecture Used

The procedure for each filter was as follows:

1. Start with a noisy color image in RGB color space.
2. Use a defined equation to convert RGB image to grey-level image.
3. Apply nonlinear filter to the image.
4. Take the original color image in RGB color space and use algorithms to convert the image into another color space (YIQ, HSV, XYZ).
5. Apply filter to the color image.
6. Convert the image back to RGB.
7. Use the same equation used in STEP 2 to convert the RGB image to gray-level.
8. Compare and contrast the grey-level images in STEPS 3 and 7.
9. Apply the signal to noise ratio to determine the image quality.

The filter algorithms were implemented using MATLAB which includes an image processing toolbox. The program is very functional and user-friendly. The MATLAB Image processing toolbox also has functions that aid in the conversion of color spaces. MATLAB is very useful for mathematical computations such as the use of matrices and vectors. There is functionality provided for matrix multiplications. The use of matrix multiplications is complex in image processing due to the use of vectors. Therefore, the magnitude, cross product, or the dot product can be used when multiplying pixel values in the equations of the filters. The use of matrix and vector multiplication is addressed in the **DISCUSSION** section.

4. RESULTS

The following table shows the Signal to Noise Ratio (SNR) for each combination.

Table 1: **RGB**

	WOS	Stack	Rank Order	Nonlinear Mean	Teager	Polynomial	Rational
Magnitude	20.8228	20.1962	14.0901	20.0767	N/A	N/A	N/A
Cross Product	N/A	N/A	N/A	N/A	7.13618	-9.05561	13.9935
Dot Product	N/A	N/A	N/A	N/A	8.70002	5.5154	9.73282

Table 2: **HSV**

	WOS	Stack	Rank Order	Nonlinear Mean	Teager	Polynomial	Rational
Magnitude	1.84821	1.8566	11.8685	1.16047	N/A	N/A	N/A
Cross Product	N/A	N/A	N/A	N/A	3.7356	-19.3505	0.73193
Dot Product	N/A	N/A	N/A	N/A	-10.8557	4.03196	-1.2355

Table 3: **YIQ**

	WOS	Stack	Rank Order	Nonlinear Mean	Teager	Polynomial	Rational
Magnitude	29.9345	29.7441	33.566	30.3338	N/A	N/A	N/A
Cross Product	N/A	N/A	N/A	N/A	7.282	-16.2347	2.9111
Dot Product	N/A	N/A	N/A	N/A	5.04037	10.4187	2.59794

Table 4: **XYZ**

	WOS	Stack	Rank Order	Nonlinear Mean	Teager	Polynomial	Rational
Magnitude	12.3291	11.967	14.4147	11.9524	N/A	N/A	N/A
Cross Product	N/A	N/A	N/A	N/A	1.14778	-5.62941	12.8802
Dot Product	N/A	N/A	N/A	N/A	0.606417	12.6513	11.5968

The following figures show the two images with the 2^{nd} highest SNR result which is the Nonlinear Mean filter using YIQ and magnitude equation.

Figure 3: Output of Gray-level image for Rank Order filter using YIQ and magnitude

Figure 4: Output of color to Gray-level image for Rank Order filter using YIQ and magnitude

Figure 5: Output of Gray-level image Nonlinear mean filter using YIQ and magnitude

Figure 6: Output of color to Gray-level image for Nonlinear mean filter using YIQ and magnitude

5. DISCUSSION

The equations used for the filters were obtained from [5]. The output equations were usually given for 2-Dimensional images. It was very simple to apply these equations to the gray level images. However, the algorithms became much more complex when it came down to applying the filters on 3-Dimensional images. One had to be careful traversing through the array of pixels since the size of the original noisy RGB image was 324 x 432 x 3. The two dimensional image size was simply 324 x 432. The 3D image has 3 channels rather than one like a 2-Dimensional image. A group of three for loops was used to manipulate the 3-Dimensional image. The loops simply computed the output using the algorithms for each of the three channels.

The multiplication of two vectors requires the use of cross product or dot product. The cross product of two vectors with 3 elements produces an output vector with three elements. However, the dot product of two vectors with 3 elements produces a scalar output, where as one value is returned. Therefore, the use of the dot product required to expand the vector to three elements in order to complete the filter algorithm. Matrix multiplication was only used for the Teager, Polynomial, and Rational filters. The Weighted Order Statistics filter, stack filter, rank order, and the Nonlinear Mean filter don't require the use of cross or dot product. In these filters, the magnitude equation was used to add and subtract the pixel values for the filter algorithms.

The magnitude equation for the RGB images was simply

$$\sqrt{R^2 + G^2 + B^2} \quad (7.0)$$

The same method was applied for the XYZ, YIQ, and HSV color spaces.

MATLAB provides a function that computes the cross product or dot product of two vectors. These functions were used in the program code. These functions were simply used whenever two pixel values were multiplied. For example, if the computation is x(m,n) * x(m+1, n +1), then the cross product and dot product functions are used separately to multiply the two values. However, the cross and dot product was not used whenever a pixel value was to a power or squared such as in the polynomial function. Instead, the power function was used because the cross product of a vector multiplied by itself is equal to zero. Therefore, using the power function would produce better results.

Nonlinear methods and techniques are intensive topics of research in image processing. The best way to find the best combination of filters with color spaces and matrix multiplication is to develop a table which has been done. From the table, one can see that the Rank order filter used on YIQ color space and magnitude for computations produced the highest Signal to Noise Ratio of 33.566. The Nonlinear mean filter used on YIQ and magnitude for computations produced the 2nd highest Signal to Noise Ratio of 30.3338. Therefore, these two combinations would be best for improving the quality of an image. From using eyesight, one can see that these two images from both combinations have great quality. From looking at the comparison table, one can see that the YIQ color space has the highest mean of the SNR when a WOS, Stack, Rank Order, or Nonlinear Mean filters are used. Again, in this case neither dot products nor cross products are used. From the table, one can conclude that the polynomial filter produces the worse SNRs when a cross product is used. The SNRs are all large negative numbers. However, the use of the dot product resulted in high positive SNRs. The highest SNRs for the three filters using cross product or dot product are above 10 dB. The use of the rational filter has the most occurrences of high SNRs. By observing the table, one can see that in general, the RGB and the XYZ color spaces produce the best results. The HSV color space clearly returns the worst results. This color space contains the most occurrences of negative SNRs.

6. CONCLUSIONS

From looking at the output of each filter, it was interesting to see the effects of edge detection and noise reduction. For example, the Teager filter seemed to be a great edge detection filter. The gray level image was pitch black with white outlines. By looking at the Signal to noise ratios from each combination, the Rank order filter on a YIQ image resulted in the highest SNR of approximately 34. The worse SNR was found in the polynomial filters in any color space with the use of cross product. Also, one can conclude that the Rank order filter has the highest mean of SNRs, which means that it is the best nonlinear filter to use compared to the others. It is noticeable that the use of the filters that use the magnitude equation produces a much larger SNR than those filters that require the use of cross or dot products.

7. REFERENCES

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