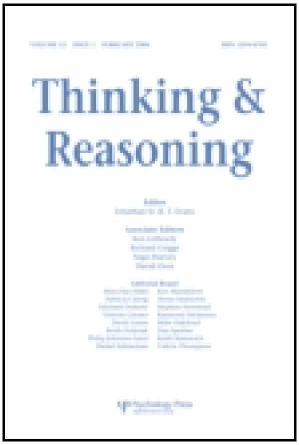

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Determining transformation distance in similarity: Considerations for assessing representational changes a priori

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Determining transformation distance in similarity: Considerations for assessing representational changes *a priori*

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The representational distortion (RD) approach to similarity (e.g., Hahn, Chater, & Richardson, 2003) proposes that similarity is computed using the transformation distance between two entities. We argue that researchers who adopt this approach need to be concerned with how representational transformations can be determined *a priori*. We discuss several roadblocks to using this approach. Specifically we demonstrate the difficulties inherent in determining what transformations are psychologically salient and the importance of considering the directionality of transformations.

Keywords: Distance, Judgement, Representation, Similarity, Transformation.

Similarity, the psychological likeness of a pair of items, is central to cognition and is often incorporated as a construct in models of cognitive processing. Most theoretical approaches to similarity are *comparison-based models* that make some assumption about the nature of mental representations and the processes that act over them. One alternative is to identify similarity as the distance between items in an *n*-dimensional representational space (Shepard, 1957). Another is to determine the features that items have in common and those that are distinct, and to allow similarity to increase with the number of common features and

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decrease with the number of distinct features (Tversky, 1977). These approaches are both intuitive, but they fail to capture the rich structure inherent in representation and the kinds of commonalities and differences that result from this structure (Gentner & Markman, 1997; Markman & Gentner, 1993). A third type of comparison model is the structural-alignment approach, which assumes that similarity depends on the degree of correspondence between the relational structures that represent the items.

One prominent structural theory is the structure mapping account, which likens similarity computation to analogical reasoning (Gentner, 1983; Gentner & Markman, 1997). Under this account, items are represented as structured systems of relational predicates, predicate arguments or roles, objects that fill those roles, and object properties. Similarity is computed by comparing these structures. Two items are similar to the extent that they share objects that play the same relational roles. One case that reveals the importance of this kind of structure in similarity is the distinction between identicality and cross-mapping. Consider Item 1 and Item 2 in Figure 1. Although the two items have the same objects with the same properties, those objects play different roles within the items' relational structure; that is, there is cross-mapping. One would therefore judge them to be similar, but not as similar as they would be if the objects in Item 2 were reversed.

An alternative to comparison theories of similarity is the representational distortion approach (RD; Hahn, Chater, & Richardson, 2003). On this view the similarity between two items is determined by the degree to which one representation would have to be distorted for it to be fully transformed into the other (Chater & Hahn, 1997; Hahn et al., 2003). In some respects RD requires some comparison (i.e., a mapping of matching components that don't require transformation) but the means of calculating similarity is different from that used by comparison models. Transformational complexity is intuitively defined as the length of the shortest computer program that could perform the transformation. This account also has an explanation for the distinction between identicality and cross-mapping: Item 2 must undergo some form of transformation to look like Item 1, while Item 1 need not be transformed at all to look like Item 1.

COMPARISON OF STRUCTURAL ALIGNMENT AND RD

In the first direct comparison of structural alignment and representational distortion theories Larkey and Markman (2005) had participants rate the similarity of object pairs like those in Figure 1. SIAM (Goldstone, 1994), a structural alignment model, provided the best qualitative fit to observed

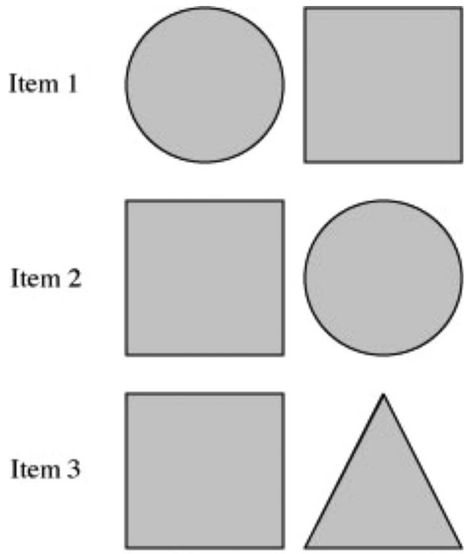

Figure 1. Stimuli like those used in Larkey and Markman (2005) and Hodgetts et al. (2009).

behaviour. However, Hodgetts, Hahn, and Chater (2009) argued that Larkey and Markman's implementation of representational distortion was not an accurate reflection of the theory. Instead of treating each observable change *in the stimuli* as a transformation, as Larkey and Markman did, they averred that an appropriate distortion model must operate on changes *in the representation of the stimuli*.

Hodgetts et al. (2009), in turn, offered a more nuanced representational distortion account, based on three types of psychologically plausible transformations between base (i.e., the item selected to be transformed into the second item in the pair) and target (i.e., the second item):

1. *Create feature* – This introduces a new feature (i.e., not present in base item) into the representation set.
2. *Apply feature* – This takes a feature in the representation set, newly created or in the base, and applies it to one or both of the existing features in the target.
3. *Swap* – This reverses individual features or whole objects in an item.

Consider Items 1 and 3 in Figure 1 with Item 1 treated as the base and Item 3 as the target. Transforming the former into the latter requires all three operations: swapping the two objects, creating a triangle feature, and applying it to the circle feature. In contrast, transforming Item 1 into Item 2 requires a single swap, reflecting the greater similarity between 1 and 2 than between 1 and 3.

To test the predictions of this model Hodgetts et al. performed two straightforward experiments, similar to those of Larkey and Markman. In Experiment 1 there were 14 unique combinations of items. In this set, shape was varied but colour was held constant, as in Items 1–3 in Figure 1. On each trial participants compared one pair of items (e.g., 1 and 2) to another pair (e.g., 1 and 3), selecting one pair as more similar than the other. The relative similarity between items in a pair was defined as the percentage of trials in which that pair was selected as more similar. As expected, items that required more transformations were perceived as being less similar than items that required fewer transformations.

ASSESSING REPRESENTATIONAL CHANGES *A PRIORI*

The disagreement between Hodgetts et al. and Larkey and Markman hinges on the extent to which researchers are capable of determining the psychological transformations thought to occur during the process of comparison. In the remainder of this manuscript we suggest that there are several major roadblocks to researchers who wish to define similarity using representational distortion, as the theory is currently formulated. First, we demonstrate that it is difficult in practice for RD to constrain the notion of a transformation *a priori* even using something as fundamental as Gestalt processing to identify which transformations are seen by participants to be psychologically salient. Second, we re-analyse the Hodgetts et al. data and present some new study data to argue that researchers need to account for which item participants are using as the base item in a pair. Lastly we conclude that RD is not currently a viable theory of similarity and discuss a computational investigation by Müller, van Rooij, and Wareham (2009) that considers the computational tractability of RD.

DETERMINING THE PSYCHOLOGICALLY SALIENT TRANSFORMATIONS

Early work on perceptual organisation by Gestalt psychologists suggests that the way points are configured can influence how these points are perceived. Using simple rows of black dots, Goldmeier (1972) demonstrated that a row of dots that are closer together is more readily perceived as a line than is a row of dots spaced further apart. Furthermore, with more complex configurations of points, the Gestalt notion of “pattern goodness” (Garner & Clement, 1963) suggests that some configurations of points are more readily perceived as patterns than others. Similarly, theories in object perception argue that objects are perceived and processed with respect to the relations among object parts (Biederman, 1987; Hummel & Stankiewicz, 1996).

EXPERIMENT 1A

Based on these Gestalt notions we used two different methods to test how participants perceive transformations. In Experiment 1a we transform configurations of points that have a high or low degree of structure with two different manipulations. In the configuration-conserved condition the overall configural structure is preserved by the combination of two transformations that independently do not preserve the configuration. In contrast, the other manipulation is not configuration preserving when a pair of transformations is combined or when the transformations are presented independently. We call these sets configuration-conserved and configuration-broken, respectively. Importantly, our configuration-conserved and configuration-broken stimuli were constructed so that the same physical transformations occur in both types of manipulations within a stimulus set.

For example, as shown in Figure 2, we expect that similarity comparisons of the configuration-broken stimuli, in which the configurations are not

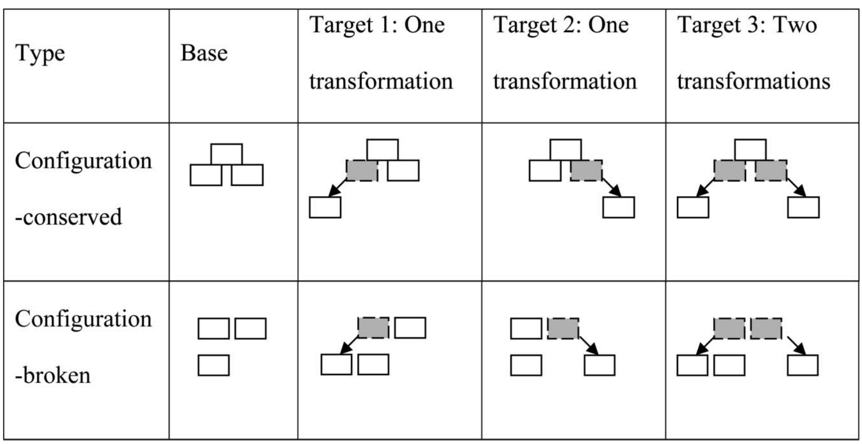

Figure 2. Sample stimulus set: configuration-conserved and configuration-broken bases were transformed one time to yield targets 1 and 2. Target 3 was created by combining the transformations.[1]

[1] Participants did not see the grey boxes or arrows; they have been added to help clarify the transformations for the reader. Furthermore, the configuration-conserved stimuli retained the same overall configural structure between the base and target 3 after two transformations whereas configuration-broken stimuli did not. Also, the same transformations were performed within targets 1, 2, and 3 for the configuration-conserved and configuration-broken bases.

preserved, will replicate the findings typical in RD studies (e.g., Hahn et al., 2003). The similarity between the base and the target stimuli should decrease as the number of transformations needed to transform the base into the target increases, making Target 3 the least similar item. In contrast, for the configuration-conserved stimuli the overall configural match is destroyed by either of the transformations alone, but preserved when the transformations are combined, making Target 3 the most similar item. Based on the work in perception, the overall configuration should be encoded and used in the similarity comparison. Thus for configuration-conserved stimuli the similarity will be lower for one transformation stimuli than for the two transformation stimuli. RD would not predict this difference between the configuration-conserved and configuration-broken stimuli *a priori*. The stimuli were constructed using exactly the same transformations within each set, thereby receiving the same number and type of counted transformations.

As shown in Figure 2, the two-transformation configuration-conserved items are critical items. Despite being constructed with two transformations, it is possible that participants will represent the change as one transformation, say expansion. Empirically, then, each stimulus would have one transformation producing identical similarity judgements. Theoretically, lower similarity judgements would suggest two transformations and higher similarity judgements would suggest one transformation. Using rated similarity to determine the number of transformations creates a circular argument. One needs to know the similarity judgements to determine the number of transformations represented, but the number of transformations needs to be determined *a priori* for RD to provide an explanation for similarity judgements. Our hypothesis for the configuration-conserved items relies on Gestalt notions, which avoids the problem of determining *a priori* how participants will respond to particular transformations and what counts as a transformation. Experiment 1a will demonstrate that it is important to understand the influence of transformations of a stimulus on people's representation of that stimulus and vice versa. The influence of the number of transformations on rated similarity depends on the structure of the stimuli.

Method

Participants. The 40 participants in Experiment 1a were undergraduates at the University of Texas at Austin. They participated to fulfil a research requirement.

Materials. The stimuli consisted of line drawings constructed in Microsoft Word using the *autoshapes* function and formed eight unique stimulus sets (see Appendix for stimuli). In Experiment 1a, within a stimulus set, there was a configuration-conserved and a configuration-broken stimulus base. As shown in the sample set in Figure 2 these bases were transformed one or two times to create the target stimuli. Each configuration-conserved stimulus had a corresponding configuration-broken stimulus that used the same one or two transformations. For example, the same transformation altered the configuration-conserved base and the configuration-broken base to form Target 1 in each set. Furthermore, following the procedure of Hahn et al. (2003), the pairs representing two transformations were constructed by combining both types of the individual transformations in one stimulus.

Procedure. Participants were presented with a booklet containing 48 pairs of stimuli. The pairs of stimuli represent the six unique base/target pairs of stimuli for the eight different stimulus sets. These pairs were randomly ordered across sets and types of pairs, subject to the constraint that members of the same set could not be presented consecutively. For each pair, the participant rated the similarity of the stimuli on a 1 to 7 scale, with 1 indicating that the stimuli were very dissimilar and 7 indicating that the stimuli were very similar.

Results

Participants rated the similarity of base/target pairs with two types of sets: configuration-conserved and configuration-broken. In addition, within the configuration-conserved and configuration-broken sets the pairs differed according to the number of transformations needed to transform the base into the target stimulus. The data were analysed using a 2 (Match Type: configuration-conserved, configuration-broken) $\times$ 2 (Transformation: 1, 2) within-participants ANOVA. All *post hoc* contrasts were Bonferroni corrected. The mean similarity ratings for pairs in each condition are shown in Figure 3.

There was a main effect for Match Type, $F(1, 39) = 260.65$, $MSE = .32$, $p < .001$, partial $\eta^2 = .87$, such that the configuration-conserved pairs ($M = 4.7$) received a higher mean similarity ratings than did the configuration-broken pairs ($M = 3.2$). In addition there was a main effect for Transformation, $F(1, 39) = 4.44$, $MSE = .34$, $p < .05$, partial $\eta^2 = .10$. The mean similarity rating for the one-transformation pairs ($M = 4.0$) was greater than the mean rating for the two-transformation pairs ($M = 3.8$).

Consistent with our predictions, there was a significant interaction between Match Type and Transformation, $F(1, 39) = 92.45$, $MSE = .18$, $p < .001$, partial $\eta^2 = .70$. This interaction reflects that for the configuration-broken stimuli, rated similarity decreased significantly from one transformation ($M = 3.6$) to two transformations ($M = 2.8$), $F(1, 39) = 69.57$, $MSE = .2$, $p < .001$, partial $\eta^2 = .64$. In contrast, for the configuration-conserved stimuli,

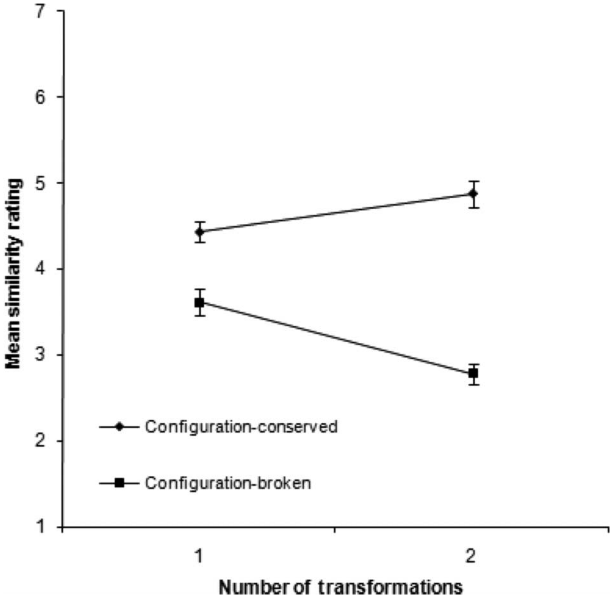

Figure 3. Mean similarity ratings for pairs of configuration-conserved and configuration-broken stimuli post one and two transformations. Error bars indicate standard errors.

rated similarity increased significantly from one transformation ($M = 4.4$) to two transformations ($M = 4.9$), $F(1, 39) = 12.67$, $MSE = .31$, $p < .01$, partial $\eta^2 = .25$.

This finding is supported by both participant and stimulus set counts. Of the 40 participants, 35 rated the configuration-broken pairs as being less similar after two transformations than after one. For the configuration-conserved pairs of stimuli a majority of participants (26 out of 40) rated the two-transformation pairs as being more similar than the one-transformation pairs, and as can be seen in the effect size for the statistical test, the increase in ratings is large. The stimulus set counts revealed that participants rated the configuration-broken stimuli in seven of the eight sets as being less similar after two transformations as compared to after one. In addition, participants rated the configuration-conserved stimuli in five of the eight stimulus sets as being more similar after two transformations as compared to after one transformation.

EXPERIMENT 1B

We present Experiment 1b to assess whether perceptual Gestalts could be integrated into RD to identify transformations *a priori*. One might argue that in Experiment 1a there is a distinct difference between transforming more-structured stimulus bases in the configuration-conserved set and less-structured bases in the configuration-broken set. According to this argument, the transformations of the configuration-conserved bases and the configuration-broken bases may actually be perceived and represented as different transformations.

In Experiment 1b the configuration-conserved stimuli from Experiment 1a were augmented to form more complex stimuli, which we call configuration-buried stimuli. For example, as shown in Figure 4, the configuration-conserved base is embedded in the centre of the configuration-buried base. Using this method the configuration-conserved bases and the configuration-buried bases had an identical set of elements that was transformed. Therefore in each case the configuration of the identical structure was conserved after two transformations of the base but not after one transformation.

We argue that, for the configuration-conserved stimuli, the overall configuration of points has been preserved between the base and two-transformation target. However, this preserved configuration is not present in a comparison of the base and the one-transformation targets. Therefore, as in Experiment 1a, we expect that as the number of transformations increases, the similarity of the items will increase.

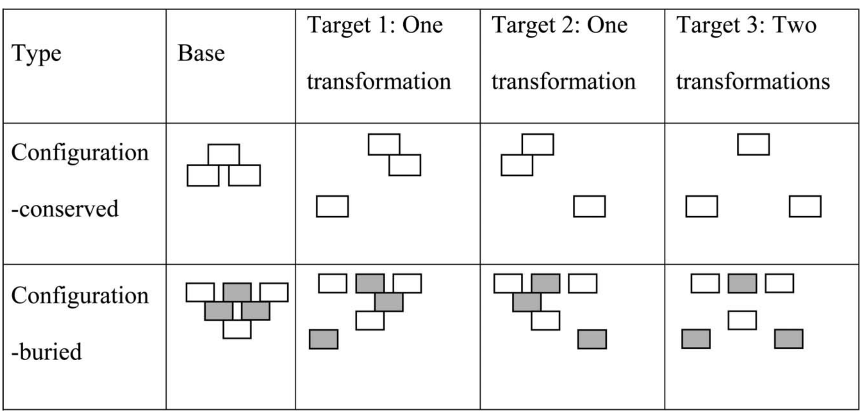

Figure 4. Sample stimulus set: configuration-conserved and configuration-buried bases were transformed one time to yield targets 1 and 2. Target 3 was created by combining the transformations.[2]

[2]The shading of the blocks in the configuration-buried stimuli depict where the configuration-conserved stimuli were embedded. Participants did not see the stimuli with this shading.

The configuration-buried stimuli also preserve the same configuration of points as the configuration-conserved stimuli. However, based on Goldmeier (1972), we expect that this matching configuration will have a reduced likelihood of being used in comparisons with these more complex stimuli, because the matching configuration does not represent a matching configuration of points for the overall figure. Instead we expect that people will focus on the changes between each component part of the configuration. As the number of transformations increases, there will be more changes among the components. Therefore we expect that the configuration-buried stimuli to be rated as less similar after two transformations than after one transformation.

Method

Participants. The 40 participants in Experiment 1b were undergraduates at the University of Texas at Austin. They participated to fulfil a research requirement, and had not previously completed Experiment 1a.

Materials and procedure. The stimuli consisted of line drawings constructed in Microsoft Word using the *autoshapes* function and formed eight unique stimulus sets (see Appendix for stimuli). In Experiment 1b, within a stimulus set, there was a configuration-conserved and a configuration-buried stimulus base. The configuration-conserved stimuli are the same as the configuration-conserved stimuli in Experiment 1a. The configuration-buried stimuli were created by doubling the number of shapes that appeared in the configuration-conserved stimuli. Furthermore, the configuration-conserved stimuli remained intact when the additional shapes were added to form the configuration-buried stimuli. As shown in the sample set in Figure 4, these bases were transformed one or two times to create the target stimuli. The exact same transformations were completed within target types. For example, the same transformation altered the configuration-conserved base and the configuration-buried base to form Target 1 in each set. Furthermore, following the procedure of Hahn et al. (2003), the pairs representing two transformations were constructed by combining both types of the individual transformations in one stimulus.

The procedure was identical to Experiment 1a.

Results

In Experiment 1b the participants rated the similarity between pairs of stimuli with two types of representations: configuration-conserved and configuration-buried. In addition the number of transformations needed to change one stimulus into the other varied. The data were analysed using a 2

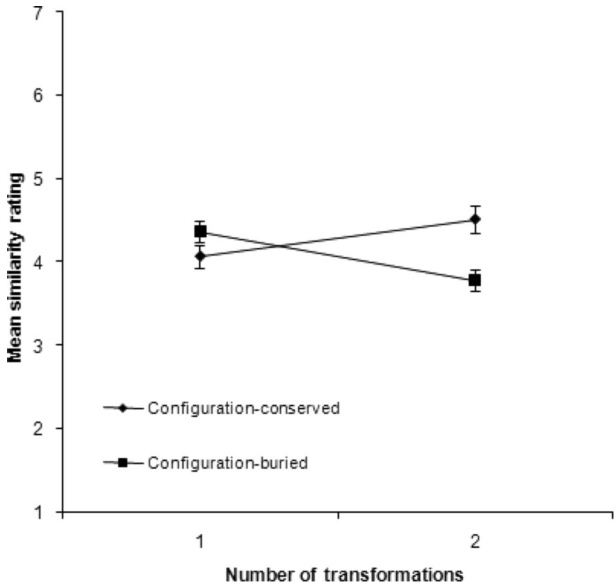

Figure 5. Mean similarity ratings for pairs of configuration-conserved and configuration-buried stimuli post one and two transformations. Error bars indicate standard errors.

(Match Type: configuration-conserved, configuration-buried) × 2 (Transformation: 1, 2) within-participants ANOVA. All *post hoc* contrasts were Bonferroni corrected such that the mean similarity ratings for pairs in each condition are shown in Figure 5.

There was a main effect for Match Type, such that the mean similarity rating of the configuration-conserved pairs ($M = 4.3$) was significantly greater than for the configuration-buried pairs ($M = 4.0$) $F(1, 39) = 6.58$, $MSE = .28$, $p < .05$, partial $\eta^2 = .14$. There was no main effect of Transformation. The mean similarity rating of the one transformation pairs ($M = 4.2$) did not differ significantly from the mean similarity rating of the two transformation pairs ($M = 4.1$) $F(1, 39) = .49$, $MSE = .21$, $p > .05$. However, there was a significant interaction between Match Type and Transformation $F(1, 39) = 46.87$, $MSE = .22$, $p < .001$, partial $\eta^2 = .55$. This interaction reflects that for the configuration-conserved stimuli, rated similarity increased from one transformation ($M = 4.1$) to two transformations ($M = 4.5$), $F(1, 39) = 9.1$, $MSE = .43$, $p < .01$, partial $\eta^2 = .19$. In contrast, for the configuration-buried stimuli, rated similarity decreased from one transformation ($M = 4.4$) to two transformations ($M = 3.8$), $F(1, 39) = 31.52$, $MSE = .22$, $p < .001$, partial $\eta^2 = .45$.

These findings are supported by both participant and stimulus set counts. Of the 40 participants a majority (26 of 40) rated the configuration-conserved stimuli as being more similar after two transformations. As in Experiment 1a this is a relatively small majority but the effect size associated with the ratings is of reasonable size. In contrast, 33 rated the configuration-buried pairs of stimuli to be less similar after two transformations than after one transformation. With respect to the stimulus sets, participants rated the similarity of the configuration-conserved pairs as being higher for five of the eight sets after two transformations. In contrast, the rated similarity of the configuration-buried pairs was higher for six of the eight sets after one transformation.

Discussion

The rated similarity of the configuration-broken and configuration-buried pairs decreased as the number of transformations between the base and target stimuli increased. In contrast, the rated similarity of the configuration-conserved pairs was higher for the two-transformation items than for the one-transformation items. These results suggest that the number of physical transformations in the stimulus domain does not necessarily correspond to the transformation distance in the mental representation domain. Moreover, it suggests that the higher similarity did not result simply from a calculation of RD using one transformation; if so, each stimulus would have been rated identically by our participants. Thus it is crucial to understand how stimulus transformations affect the underlying mental representations of the items and to determine what counts as a meaningful transformation. Our studies suggest that it is difficult to predict *a priori* what would count as one or two transformations. This problem worsens as the number of possible transformations increases. As such, RD may not be able to rely on perceptual Gestalts to gain traction on determining the number of transformations *a priori*.

CONSIDERING WHICH ITEM TO USE AS THE BASE ITEM FOR TRANSFORMATION

In this section we consider the importance of determining which item is psychologically treated as the base item during a similarity judgement (Tversky, 1977). Using RD the assumption is that the number of transformations required to turn the "base" into the target is used. In this approach the item considered to be the base is the item that produces the fewest transformations. Table 1 describes the stimuli and results from Experiment 1 in Hodgetts et al. (2009). Schematic descriptions of the items appear in the first and second column. Each item is described with two

TABLE 1
Hodgetts et al. (2009) Experiment 1

Pair 1	*Pair 2*	*Percentage selected*	*# of transformations (Hodgetts et al.)*	*# of transformations (Pair 1 base)*
AA	AA	91.56	0	0
AB	AB	73.67	0	0
AB	BA	69.39	1	1
AA	AB	51.12	2	2
AA	BA	49.65	2	2
AB	AA	47.63	2	1
AB	BB	49.64	2	1
AA	BB	61.99	2	2
AB	AC	51.09	2	2
AB	CB	41.82	2	2
AB	BC	36.7	3	3
AB	CA	34.38	3	3
AA	BC	29.45	4	4
AB	CC	23.57	4	2
			$r = -.95$	$r = -.80$

letters, corresponding to unique shape features. For example, Item 1 in Figure 1 would correspond to AB, while Item 3 would be BC.

The selection percentages (i.e., relative similarity) and the number of transformations between items in a pair, as defined by Hodgetts et al., appear in the third and fourth columns of Table 1, respectively. Overall there was a strong negative correlation between similarity and number of transformations ($r = -.95$, $p < .01$). This correlation was significantly stronger than the –.76 correlation for SIAM ($z = 1.88$, $p < .05$) and the –.74 correlation for Falkenhainer, Forbus, and Gentner's (1989) Structure Mapping Engine (SME) ($z = 1.98$, $p < .05$). Hodgetts et al. emphasise that this superior fit is particularly impressive because their transformation model—unlike SIAM and SME—has no free parameters.

This impressive correlation for the transformation model is largely due to assumptions about which pair is considered to be the base pair. In Hodgetts et al.'s formulation the first step is to calculate the number of transformations required to convert each item into the other. The item requiring the greater number of transformations is then selected as the base. Although it is plausible that participants compare items in this manner, it is more likely that people use the item that is presented first as the base. The items highlighted in grey in Table 1 are those items when Pair 2 instead of Pair 1 was used as the base pair by Hodgetts et al. for their calculations. When the number of transformations is calculated by using Pair 1 as the base for all items, the correlation drops to –.80, indistinguishable from SIAM ($z = .23$, *ns*) and SME ($z = .34$, *ns*).

EXPERIMENT 2

To consider this possibility we performed a simple experiment in which we varied the position of the items on the screen. On each trial participants saw two items and were asked to make a similarity judgement. Each critical trial contained a red circle and a second item created by transforming the red circle. On half of the trials the red circle appeared on the left side of the screen, and on the other half the red circle appeared on the right. We manipulated screen position as a way to influence the item treated as the base during the similarity comparison. Our participants were native English speakers, and reading from left to right automatically could cause participants to select the leftmost item as the base. To increase this possibility we included words in our stimuli: [Shape A] is like [Shape B]. We used simple transformations: expansion, contraction, indention, and protrusion. Our selection of transformations was inspired by Leyton (1989), who argued that people infer the causal history of an object based on the presence of two features: curvature and symmetry. When encountering pairs of objects that appear to be the same object but at different stages, he further argues that people can infer the intervening-process history. RD would predict no difference in similarity judgements based on screen position for items with identical transformations because it is the number of transformations that determines similarity.

Method

Participants. The 50 participants in Experiment 2 were undergraduates at the University of Texas at Austin. They participated to fulfil a research requirement.

Materials. The stimuli consisted of shapes constructed in Serif DrawPlus. There were three transformations of each type: expansion, contraction, protrusion, and indention. For example, for expansion the transformations referred to below as 1, 2, and 3 were the base circle

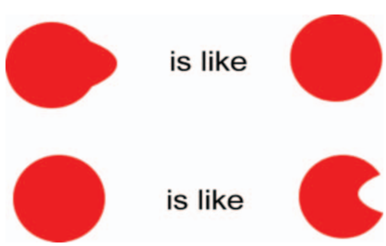

Figure 6. Two example trials from Experiment 2.

expanded by $^{1}/_{4}$ inch, expanded by $^{1}/_{2}$ inch, and expanded by $^{3}/_{4}$ inch. The expansion and contraction stimuli were created using the same method, and the protrusion and indention mirrored each other. For example, as shown in Figure 6, the 1-indention item was created by inverting the 1-protrusion item. As such, the difference in transformation between a 1-indention item and the base and the 1-protrusion item and the base is identical.

Procedure. Participants completed 262 similarity judgements in the experiment for pairs of items presented on a computer screen. Of these 24 are considered critical trials with the remainder being filler items. Item presentation was randomised for each participant. The critical stimuli represent the six unique base/target pairs of stimuli for the four different stimulus sets. That is, each transformation type has three forms and the circle was presented on both the right and the left. For each pair the participant rated the similarity of the stimuli on a 1 to 9 scale, with 1 indicating that the stimuli were very dissimilar and 9 indicating that the stimuli were very similar.

Results

The data were analysed using a 2 (Circle Position: left, right) × 4 (Transformation Type: expansion, contraction, indention, protrusion) × 3 (Transformation: 1, 2, 3) within-participants ANOVA. There was a main effect of Transformation Type, $F(3, 66) = 94.34$, $MSE = 3.32$, $p < .001$, partial $\eta^2 = .81$, and a main effect of Transformation, $F(2, 44) = 62.89$, $MSE = .22$, $p < .001$, partial $\eta^2 = .74$. There was no main effect of Circle Position. As the number of transformations increased, the rated similarity decreased ($M = 6.4$, 5.6, and 5.0 for 1, 2, and 3 transformations, respectively) showing a strong linear trend, $F(1, 22) = 88.69$, $MSE = 2.04$, $p < .001$, partial $\eta^2 = .80$, and participants' ratings for the protrusion and indention items did not differ ($M = 4.4$ and 4.6, respectively), $t(45) = 1.33$, $p = .19$. However, participants rated the expansion items as having greater similarity with the base ($M = 7.4$) as compared to the contraction items ($M = 6.6$), $t(45) = 5.36$, $p < .001$, $d = .80$.

There was also a Transformation Type × Circle Position interaction, $F(3, 66) = 3.89$, $MSE = 1.12$, $p = 0.013$, partial $\eta^2 = .15$. There were no other significant interaction effects. For Expansion, items were rated as more similar when the circle appeared on the right side of the screen ($M = 7.5$) than the left side of the screen ($M = 7.2$), $F(1, 24) = 8.02$, $MSE = .48$, $p = .009$, partial $\eta^2 = .25$, while there was no difference for Contraction ($M = 6.6$ and 6.6), $F = .16$. For Protrusion, items were rated as more similar when the circle appeared on the left side of the screen ($M = 4.8$) than the right side of the screen ($M = 4.2$), $F(1, 24) = 6.19$, $MSE = 1.7$, $p = .021$, partial $\eta^2 = .21$, while there was no difference for Indention ($M = 4.3$ and 4.4), $F = .21$.

Discussion

By recoding the stimulus transformation in Hodgetts et al., and performing Experiment 2, we demonstrate that it is critical to realise that the base item used by participants fundamentally affects similarity judgements. The expand and contract items had exactly the same transformations (e.g., $^1/_4$ inch smaller than the base or $^1/_4$ inch larger than the base in the 1 contract and 1 expand stimuli). However, participants' treatment of this transformation differed. Participants rated expansion-transformed items as being more similar to the circle than contraction-transformed items. Moreover the position of the circle on the screen, which could influence the item selected by participants as the base (as native English speakers our participants automatically read from left to right), also interacted with the type of transformation to alter similarity judgements. Our results suggest that adding and subtracting a feature from the base are also treated differently. RD cannot account for these effects because the number of transformations alone determines similarity. As suggested by Hahn et al. (2003), RD does allow for different transformation weights and therefore could theoretically apply higher weights to transformations using a specific item as a base. Based on our data, however, it is not clear how this unequal weighting could reasonably be applied *a priori*. Similarity judgements varied with the type of transformation and position of the circle base. Furthermore, allowing unequal weights to a transformation and its inverse seems very unnatural, and RD would no longer be void of free parameters. It will be critical for RD to account for base item selection in accounting for similarity judgements.

CONCLUSION

The representation distortion approach is not a viable theory of similarity in its current formulation. We do believe that RD has made a conceptual contribution to the study of similarity because it has challenged researchers to think about similarity processing in new ways. However, we believe that theories of similarity and tests of those theories must make meaningful *a priori* claims about representation. It is important that theories of similarity can identify psychologically meaningful transformations and account for the item selected as the base item that is transformed. In support of this view participants in Experiment 1a rated the similarity of pairs of configuration-conserved and configuration-broken stimuli after one or two transformations. Overall, participants rated the configuration-conserved pairs to be more highly similar than the configuration-broken pairs. In addition, as compared to the similarity ratings after one transformation, participants rated the configuration-broken stimuli as being less similar and rated the configuration-conserved stimuli as being more similar after two transformations. In Experiment 1b the similarity ratings of the configuration-conserved stimulus pairs again increased as the number of transformations increased. In contrast, like the configuration-broken stimuli in Experiment 1a, the similarity ratings of the configuration-buried stimulus pairs decreased as the number of transformations increased. The results from these experiments highlight the potential importance of transformation distance in the similarity comparison process, while recognising that it is crucial to understand and predict the influence of transformations on item representations.

In Experiment 2 we demonstrated that researchers do need to account for which item is considered to be the base item during the process of computing similarity. Items with identical transformations were not rated as having identical similarity with the circle item and the placement of the circle item on the screen influenced judgements for half of the transformations. One avenue for future research could be examining when and why base selection matters.

Lastly, while the Hodgetts et al. model has no numerical free parameters, the idea of transformations is sufficiently vague to allow a great deal of flexibility. All of the possible transformation operations are essentially hidden degrees of freedom. These hidden degrees of freedom can lead to apparent statistical anomalies (McKay, Bar-Natan, Bar-Hillel, & Kalal, 1999). Model flexibility per se is not necessarily a problem, but that flexibility should be explicitly built in to the model. Implicit flexibility can lead to inappropriate comparisons between models, domain rigidity, and coding errors in some cases. Furthermore it is not necessarily true that the computational processes required to implement RD in one domain (e.g., computers) are the same processes required to implement RD in human cognition.

Müller et al. (2009) explicitly tested the computational tractability of RD. They formulated four different computational versions of RD and discovered that computing similarity using each formulation required super-polynomial time algorithms. As explained by Müller et al., super-polynomial algorithms are considered to be intractable because they take a very long time unless there are only a small number of inputs. To address this intractability Müller et al. examined specific parameters to determine the source of the intractability. Some of these parameters included the length of the shortest sequence of transformations, the size of the set of possible transformations, the maximum lengths of the representations of both stimuli, the maximum lengths of all of the intervening representations required to turn one representation into another, and the size of the set of possible transformations most likely to be relevant in the current context. Müller et al. find that RD is tractable only when researchers assume that similarity is only calculated for items that are somewhat similar, the set of transformations used in a specific context is relatively small, and that the number of total transformations cannot be too large. RD theorists could potentially allow for a non-computational process to select the set of transformations but this would reduce the explanatory power of RD (van Rooij, 2008). RD theorists recognise that the set of transformations needs to be rather large to account for the data (Hahn et al., 2003), because there needs to be a large enough set of possible transformations from which subsets of relevant transformations can be generated. However, the requirement of such a large set of possible transformations confronts the issues raised by Müller et al. that RD needs a smaller set of possible transformations to be computationally tractable. Thus on the one hand RD needs a large set of possible transformations to provide sufficient degrees of freedom to fit the data, but on the other hand the theory is computationally intractable with large numbers of transformations.

We suggest that RD theorists should focus on considering how to constrain the notion of a transformation and to recognise that people likely select a particular item to use as a base during the similarity computation process. The current operationalisation does not provide enough specificity to mirror similarity processing. As an example, we consider some hypotheses suggested by structural alignment theory. Derived from structure mapping theory (Gentner, 1983), structural alignment assumes that the similarity between two items results from both the commonalities and the differences that emerge from comparisons (Gentner & Markman, 1997) and this approach to similarity has been implemented by a variety of symbolic and connectionist models (Falkenhainer et al., 1989; Holyoak & Thagard, 1989; Hummel & Holyoak, 1997; Keane, Ledgeway, & Duff, 1994; Love, 2000). The theory assumes that items are represented by structured hierarchical representations consisting of object *attributes*, which describe objects, and *relations*, which relate two or more object attributes or relations (Gentner, 1983; Gentner & Markman, 1997; Markman, 2001). During a similarity comparison, items are placed into correspondence that conserves their commonalities (Markman, 1999).

By applying the predictions of structural alignment to representational distortion, the notion of a transformation may be constrained. For example, transformations of attributes may be perceived to be qualitatively different and independent from perceptions of transformations of relations (Goldstone, Medin, & Gentner, 1991). Furthermore, dimensions like size and colour may be quantified over entities defined as shapes so transformations on these attributes may be treated differently than transformations on shapes (Love & Markman, 2003).

A hybrid model combining structural alignment and RD could potentially use RD principles to select the best mapping of items when multiple mappings result from the comparison process. The best mapping would be the one that involves the lowest complexity to turn the representation of one item into the representation of the other. That said, if RD theorists are interested in creating such a model it will be important to demonstrate empirically that the mappings created and the final mapping selected by a structural alignment model, such as SME, are not the mappings generated and selected by people, and therefore that RD principles could contribute to the process. We hope that RD theorists further constrain the transformation process to allow this framework to influence the body of work on similarity judgement.

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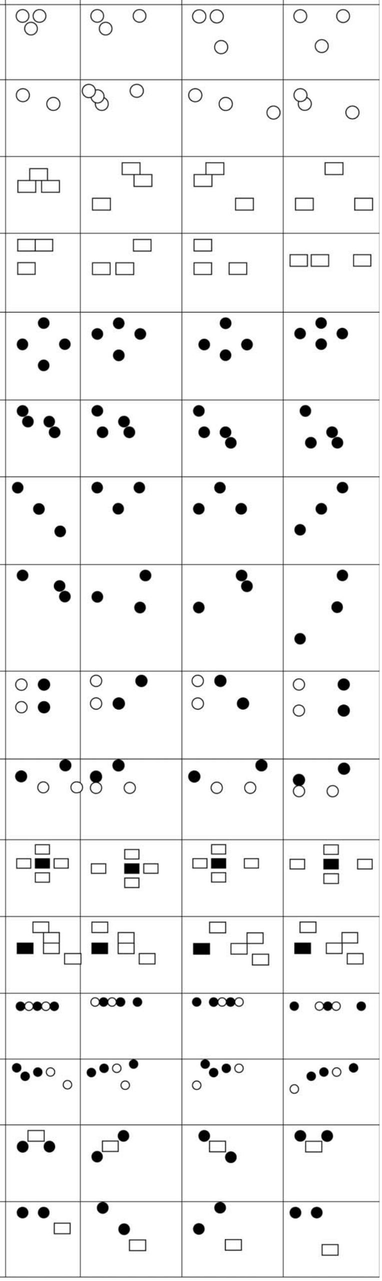
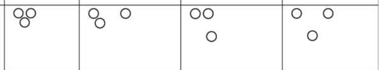
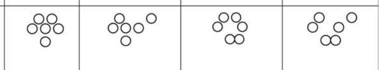
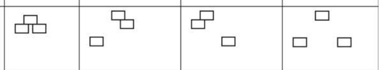
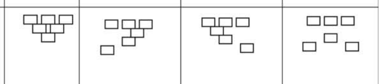
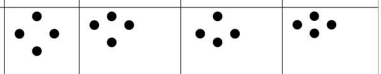
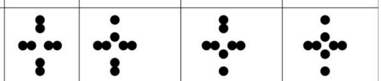
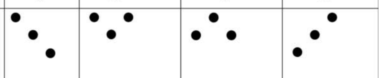
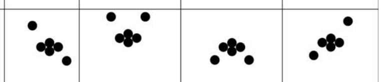
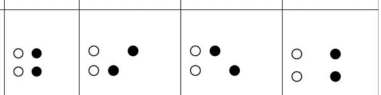
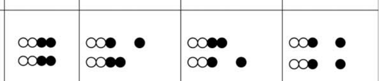
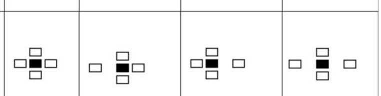
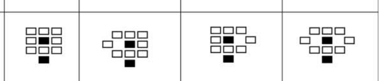
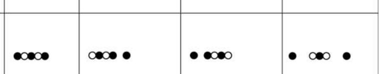
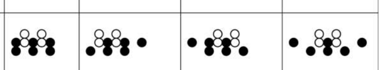
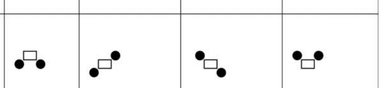
Appendix

Stimuli from Experiment 1a

Stimulus set	Type	Base	Target 1	Target 2	Target 3
1	Configuration-conserved				
	Configuration-broken				
2	Configuration-conserved				
	Configuration-broken				
3	Configuration-conserved				
	Configuration-broken				
4	Configuration-conserved				
	Configuration-broken				
5	Configuration-conserved				
	Configuration-broken				
6	Configuration-conserved				
	Configuration-broken				
7	Configuration-conserved				
	Configuration-broken				
8	Configuration-conserved				
	Configuration-broken				



Stimuli from Experiment 1b

Stimulus set	Type	Base	Target 1	Target 2	Target 3
1	Configuration-conserved				
	Configuration-buried				
2	Configuration-conserved				
	Configuration-buried				
3	Configuration-conserved				
	Configuration-buried				
4	Configuration-conserved				
	Configuration-buried				
5	Configuration-conserved				
	Configuration-buried				
6	Configuration-conserved				
	Configuration-buried				
7	Configuration-conserved				
	Configuration-buried				
8	Configuration-conserved				
	Configuration-buried